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In 2006 all ECB publications will feature a motif taken from the €5 banknote.

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Abstract

This paper assesses the sources of volatility persistence in Euro Area money market interest rates and the existence of linkages relating volatility dynamics. The main findings of the study are as follows. Firstly, there is evidence of stationary long memory, of similar degree, in all series. Secondly, there is evidence of fractional cointegration relationships relating all series, except the overnight rate. Two common long memory factors are found to drive the temporal evolution of the volatility processes. The first factor shows how persistent volatility shocks are trasmitted along the term structure, while the second factor points to excess persistent volatility at the longer end of the yield curve, relative to the shortest end. Finally, impulse response analysis and forecast error variance decomposition point to forward transmission of shocks only, involving the closest maturities.

Keywords: *Money market interest rates; liquidity effect; realized volatility; fractional integration and cointegration, fractional vector error correction model.*

JEL classification: C32; F30; G10

Non-technical summary

Recent work by Prati et al. (2003), Bartolini and Prati (2003a), and Bartolini et al. (2002) document the close connection between the institutional details of the operational frameworks for the implementation of monetary policy by central banks and the behavior of overnight interest rates in the US, the euro area and other G-7 countries. Moreover, Bartolini and Prati (2003b) show that short-term interest rate volatility also reflects differences in central banks' commitment to interest rate smoothing at high frequency. Their evidence is supported by other empirical papers showing the relevance of institutional details in shaping the behavior of money market rates and their volatility (see Ayuso et al., 1997; Hartmann et al., 2001). Cassola and Morana (in press) estimate the factors underlying the volatility of the overnight interest rate and its transmission along the euro money market yield curve using hourly data. The estimates show repetitive intradaily and monthly patterns that can be explained by the microstructure of the money market and the institutional features of the Eurosystem's operational framework for monetary policy implementation. Strong persistence is detected in all log-volatility processes, and two common long-memory factors are extracted. The first factor explains the long-memory dynamics of the shortest maturities, while the second factor explains the transmission of volatility along the money market yield curve. The present paper complements and extends the study of Cassola and Morana (in press) by considering a new econometric methodology, a different log-volatility measure (the realized volatility estimator) and an extended and improved data set. Moreover, in the present paper the analysis considers both the short-run and long-run properties of the data, with cointegration analysis performed by means of the frequency domain principal components approach of Morana (2004a, 2005) (see also Beltratti and Morana, 2006), and the short-run dynamics estimated by means of a fractional vector error correction model (F-VECM). Additionally, a new approach to permanent-persistent-non persistent decomposition is implemented. The latter decomposition also allows to shed light on the issue on whether the change in the monetary policy operational framework carried out by the European Central Bank (ECB) in March 2004 has had any impact on money market volatility. Finally, in the present paper the transmission of volatility shocks is investigated by means of impulse response analysis and forecast error variance decomposition. Hence, in addition to the modelling tools employed, the novelty of the paper is also in the empirical analysis, providing evidence on issues so far unexplored for the euro area money market.

The empirical approach followed in the paper is broadly consistent with

the modelling of the term structure of interest rates proposed by Piazzesi (2001), which generates a money market volatility curve that is U-shaped. Intuitively, such shape can be explained by two different features. First, liquidity noise and other money-market disturbances are shocks to the spread between the overnight interest rate and the central bank target rate (in general a short-term rate). These shocks affect the term structure of interest rates only at its very short end. Second, the spread between the target rate and the two-year interest rate reflects the direction of expected future short-term interest rates and, thus, is mainly affected by news about the cyclical evolution of the economy and the perception, by market participants, of how the central bank will "react" to such news. These news affect longer-term rates more intensely than they affect intermediate interest rates. This is due to the fact that markets expect the central bank, when changing short-term interest rates, to move in a sequence of small steps in one direction, and these changes to be somewhat persistent. Thus, the shape of the volatility curve is also related to the expected timing of official interest rate decisions and the "inertia" in policy interest rates. However, in Piazzesi (2001) neither the yields nor the volatility processes for short-term money market interest rates display long memory.

Modelling the term structure of interest rates with fractionally cointegrated yields seems to be a largely unexplored territory. One of the few exceptions is Backus and Zin (1993), which develop a term structure model where yields are driven by the fractionally integrated short-term interest rate process. Backus and Zin (1993) argue that the volatility curve implied by this process matches very closely the *average* volatility curve observed in U.S. government bond data for the post war period. However, in their model the volatility processes are non-stochastic. In contrast, we find ample evidence that interest rate volatility in the euro money market is stochastic and strongly persistent. As pointed out by Borio and McCauley (1996), there is pervasive evidence of persistence of volatility in bond markets, i.e. volatility shocks tend to exercise significant effects for longer than what is expected for a short memory process[1], showing a slow hyperbolic pattern of decay, rather than a quick exponential one. Three main interpretations have been suggested for this phenomenon. The first interpretation is that volatility reacts immediately on the arrival of information, however the arrival of "news" itself exhibits persistence (Granger, 1980; Andersen and Bollerslev, 1997). The second interpretation is that market participants respond at various speeds to information that arrives uniformly over time or to different volatility dynamics (high frequency versus low frequency; Muller et al., 1997), thus generating

[1] For instance an I(0) process.

persistence. The third one is that persistence reflects the memory of risk-averse traders. In the paper, the "long memory" property of volatility is accurately investigated and modelled, also by employing testing approaches which are robust to the presence of such a feature.

The main findings of the study are as follows. Firstly, there is evidence of long memory in the money market interest rates volatility processes, with two common long memory factors driving the temporal evolution of the series analyzed. The identified cointegration space reveals near homogeneous bivariate relationships involving the closest maturities. Moreover, the common long memory factor analysis points to a single factor explaining the transmission of persistent volatility shocks along the term structure, and to a second factor explaining excess persistent volatility at the longer end of the yield curve, relative to the shortest end. Forward propagation of persistent volatility shocks is also pointed out by the impulse response analysis and the forecast error variance decomposition, while no evidence of forward transmission of liquidity shocks is found.

1 Introduction

Recent work by Prati et al. (2003), Bartolini and Prati (2003a), and Bartolini et al. (2002) document the close connection between the institutional details of the operational frameworks for the implementation of monetary policy by central banks and the behavior of overnight interest rates in the US, the euro area and other G-7 countries. Moreover, Bartolini and Prati (2003b) show that short-term interest rate volatility also reflects differences in central banks' commitment to interest rate smoothing at high frequency. Their evidence is supported by other empirical papers showing the relevance of institutional details in shaping the behavior of money market rates and their volatility (see Ayuso et al., 1997; Hartmann et al., 2001). Cassola and Morana (in press) estimate the factors underlying the volatility of the overnight interest rate and its transmission along the euro money market yield curve using hourly data. The estimates show repetitive intra-daily and monthly patterns that can be explained by the microstructure of the money market and the institutional features of the Eurosystem's operational framework for monetary policy implementation. Strong persistence is detected in all log-volatility processes, and two common long-memory factors are extracted. The first factor explains the long-memory dynamics of the shortest maturities, while the second factor explains the transmission of volatility along the money market yield curve. The present paper complements and extends the study of Cassola and Morana (in press) by considering a new econometric methodology, a different log-volatility measure (the realized volatility estimator) and an extended and improved data set. Moreover, in the present paper the analysis considers both the short-run and long-run properties of the data, with cointegration analysis performed by means of the frequency domain principal components approach of Morana (2004a, 2005) (see also Beltratti and Morana, 2006), and the short-run dynamics estimated by means of a fractional vector error correction model (F-VECM). Additionally, a new approach to permanent-persistent-non persistent decomposition is implemented. The latter decomposition also allows to shed light on the issue on whether the change in the monetary policy operational framework carried out by the European Central Bank (ECB) in March 2004 has had any impact on money market volatility. Finally, in the present paper the transmission of volatility shocks is investigated by means of impulse response analysis and forecast error variance decomposition. Hence, in addition to the modelling tools employed, the novelty of the paper is also in the empirical analysis, providing evidence on issues so far unexplored for the euro area money market.

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The paper is organized as follows. In Sections 2 and 3 the econometric methodology and the data employed in the study are presented; in Section 4 the empirical results of the study are discussed, while in Section 5 conclusions are drawn. Finally, methodological details and the results of a Monte Carlo exercise aiming to assess the econometric tools proposed in the paper are included in the Appendix.

2 Econometric methodology

Consider the following decomposition for the n-variate volatility process $\mathbf{y}_t$

$$\mathbf{y}_t = \mathbf{\Xi b}_t + \mathbf{\Theta} \boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_t,$$

where $\mathbf{b}_t$ is a k-variate deterministic common structural break process, $\boldsymbol{\mu}_t$ is a s-variate stationary common long memory process ($I(d)$, $0 < d < 0.5$)[2], $\boldsymbol{\varepsilon}_t$ is a n-variate residual idiosyncratic $I(0)$ component, $\mathbf{\Xi}$ is the $n \times k$ common break processes loading matrix, and $\mathbf{\Theta}$ is the $n \times s$ common long memory processes loading matrix.

Following Morana (2006b), estimation of the common factor model is achieved in two steps.

Firstly, a permanent-persistent-non persistent (P-P-NP) decomposition is carried out for any of the n volatility processes, i.e.

$$y_{i,t} = b^*_{i,t} + \mu^*_{i,t} + \varepsilon^*_{i,t}, \quad i = 1, ..., n,$$

[2] See Baillie (1996) for an introduction to long memory processes.

where $b^*_{i,t}$, the permanent component, is estimated by $\hat{b}^*_{i,t}$, computed either by means of the flexible least squares approach or the adaptive semiparametric approach described in Appendix A, and $\mu^*_{i,t}$ and $\varepsilon^*_{i,t}$, the persistent and non persistent components, respectively, are estimated by $\hat{\mu}^*_{i,t}$ and $\hat{\varepsilon}^*_{i,t}$, computed by means of the Fourier transform method of Morana (2006b), also described in Appendix A.[3] Monte Carlo results reported in Morana (2006b) and in Appendix B strongly support the proposed filtering methodology, pointing to its robustness to the degree of persistence of the actual series, sample size, presence of observational noise and biased estimation of the fractional differencing parameter.

Secondly, the common break and long memory processes are estimated by means of a principal components (PCA) approach, implemented using the estimated permanent and persistent components, as follows

$$\mathbf{y}_t = \hat{\mathbf{\Xi}}\hat{\mathbf{b}}_t + \hat{\mathbf{\Theta}}\hat{\boldsymbol{\mu}}_t + \hat{\boldsymbol{\varepsilon}}_t,$$

where $\hat{\mathbf{\Xi}} = \hat{\mathbf{A}}\hat{\mathbf{\Lambda}}_b^{1/2}$ is the estimated $n \times k$ common break processes loading matrix, $\hat{\mathbf{\Lambda}}_b$ is the diagonal matrix of the estimated non zero eigenvalues of the reduced rank variance-covariance matrix of the (estimated) break processes $\mathbf{\Sigma}_{b^*}$ (rank $k < n$), $\hat{\mathbf{A}}$ is the matrix of the associated orthogonal eigenvectors, and $\hat{\mathbf{b}}_t = \hat{\mathbf{\Lambda}}_b^{-1/2}\hat{\mathbf{A}}'\mathbf{b}^*_t$ is the $k \times 1$ vector of the standardized ($\mathbf{\Sigma}_b = \mathbf{I}_k$) principal components or common break processes. Moreover, $\hat{\mathbf{\Theta}} = \hat{\mathbf{B}}\hat{\mathbf{\Lambda}}_\mu^{1/2}$ is the estimated $n \times s$ common break processes loading matrix, $\hat{\mathbf{\Lambda}}_\mu$ is the diagonal matrix of the non zero eigenvalues of the reduced rank variance-covariance matrix of the (estimated) long memory components $\mathbf{\Sigma}_{\mu^*}$ (rank $s < n$), $\hat{\mathbf{B}}$ is the matrix of the associated orthogonal eigenvectors, and $\hat{\boldsymbol{\mu}}_t = \mathbf{\Lambda}_\mu^{-1/2}\hat{\mathbf{B}}'\boldsymbol{\mu}^*_t$ is the $s \times 1$ vector of the estimated standardized ($\mathbf{\Sigma}_\mu = \mathbf{I}_s$) principal components or common long memory processes. Finally, $\hat{\boldsymbol{\varepsilon}}_t = \hat{\boldsymbol{\varepsilon}}^*_t + \hat{\boldsymbol{\varepsilon}}_{b,t} + \hat{\boldsymbol{\varepsilon}}_{\mu,t}$, where $\hat{\boldsymbol{\varepsilon}}_{b,t}$ is an $n \times 1$ vector of idiosyncratic components from $\mathbf{\Xi}\mathbf{b}_t = \hat{\mathbf{\Xi}}\hat{\mathbf{b}}_t + \hat{\boldsymbol{\varepsilon}}_{b,t}$ and $\hat{\boldsymbol{\varepsilon}}_{\mu,t}$ is an $n \times 1$ vector of idiosyncratic components from $\boldsymbol{\mu}_t = \hat{\mathbf{\Theta}}\hat{\boldsymbol{\mu}}_t + \hat{\boldsymbol{\varepsilon}}_{\mu,t}$.[4]

[3] Note that the break process $b^*_{i,t}$, the permanent component, can be interpreted as the long-run forecast for the series $y_{i,t}$, since $\lim_{s\to\infty} E_{t+s}y_{i,t} = b^*_{i,t+s}$, given that for $d < 0.5$ $\lim_{s\to\infty} E_{t+s}\mu^*_{i,t} = 0$, and for $b < 0.5$ $\lim_{s\to\infty} E_{t+s}\varepsilon^*_{i,t} = 0$. On the other hand, the persistent component, i.e. the long memory component, can be interpreted as the medium-run forecast, since for a sufficiently long but finite forecast horizon $\lim_{s\to k<\infty} E_{t+s}(y_{i,t} - b^*_{i,t}) = \lim_{s\to k<\infty} E_{t+s}\mu^*_{i,t}$, since $\lim_{s\to k<\infty} E_{t+s}\varepsilon^*_{i,t} = 0$.

[4] See Morana (2006) also for an extension of the procedure to the non stationary long memory case ($0.5 < d < 1$) and to the case of multiple common degrees of persistence.

The above principal components approach has been originally proposed by Bierens (2000)[5] for the estimation of common deterministic break processes, and extended to the estimation of common long memory processes by Morana (2006). The validity of the proposed estimation procedure has been shown by Bai (2004, 2003) and Bai and Ng (2001). In fact, recent theoretical developments of Bai (2004, 2003) and Bai and Ng (2001) have justified the use of the PCA estimator also for dependent processes. In particular, Bai (2004) has considered the generalization of PCA to the case in which the series are weakly dependent processes, establishing consistency and asymptotic normality when both the unobserved factors and idiosyncratic components show limited serial correlation, also allowing for heteroskedasticity in both the time and cross section dimension in the idiosyncratic components. In Bai (2003) consistency and asymptotic normality has been derived for the case of I(1) unobserved factors and I(0) idiosyncratic components, also in the presence of heteroskedasticity in both the time and cross section dimension in the idiosyncratic components. Finally, Bai and Ng (2001) have established consistency also for the case of I(1) idiosyncratic components. As pointed out by Bai and Ng (2001) consistent estimation should also be achieved by PCA in the intermediate case represented by long memory processes. Monte Carlo results supporting the use of the PCA approach also in the case of long memory processes, and the proposed econometric methodology, have been provided in Morana (2006b). It is shown that the performance of the principal components approach is indeed not affected by the presence of long memory, performing well independently of the degree of persistence, the sample size, and the number of factors, being also robust to the presence of moderate noise.

3 The data

The data employed in this study are 5-minute observations for the overnight interest rate, and the one-week, two-week, one-month, three-month, six-month, nine-month and twelve-month EONIA swap rates. Intra-daily observations have been computed as averages of real-time, bid-ask quotes taken from REUTERS screens. The raw data were filtered for typing errors and other outliers using the simple and the dynamic filtering techniques explained in Brousseau (2005).[6] It is known that these quotes are only indicative. Yet, like Hartmann et al. (2001) it is assumed that there is a close link be-

[5] This result can not be found in the published version of Bierens (2000) paper.

[6] The authors are grateful to Vincent Brousseau, in the Market Operations Analysis Division, European Central Bank, for providing the filtered data.

tween market activity and quotes updating. The sample is from 28/11/2000 through 22/04/2005, for a total of 107291 5-minute observations, excluding weekends and holidays. Excluding thin trading days (days in which prices did not change) 91392 usable observations are left, i.e. 952 days, with 95 five-minutes observations each (from 9 a.m. to 5 p.m.).

3.1 The variance process

The daily variance process has been computed by means of the realized variance estimator. Following Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002), suppose that the log $M \times 1$ vector price process, p_t, follows a multivariate continuous-time stochastic volatility diffusion

$$dp_t = \mu_t dt + \Omega_t dW_t,$$

where W_t denotes a standard M-dimensional Brownian motion process, and both the processes for the $M \times M$ positive definite diffusion matrix Ω_t and the M-dimensional instantaneous drift μ_t are strictly stationary and jointly independent of the W_t process.

Then, conditional on the sample path realization of Ω_t and μ_t, the distribution of the continuously compounded h-period return

$$r_{t+h,h} = p_{t+h} - p_t$$

is

$$r_{t+h,h} | \sigma \left\{ \mu_{t+\tau}, \Omega_{t+\tau} \right\}_{\tau=0}^{h} \sim N\left(\int_0^h \mu_{t+\tau} d\tau, \int_0^h \Omega_{t+\tau} d\tau\right).$$

The integrated diffusion matrix

$$\int_0^h \Omega_{t+\tau} d\tau$$

can be employed as a measure of multivariate volatility.

By the theory of quadratic variation, under some weak regularity conditions,

$$\hat{\Omega}_{t+h} = \sum_{j=1,\ldots,[h/\Delta]} r_{t+j\cdot\Delta,\Delta} r'_{t+j\cdot\Delta,\Delta} \xrightarrow{p} \int_0^h \Omega_{t+\tau} d\tau,$$

i.e. the realized variance covariance matrix estimator is a consistent estimator, in the frequency of sampling ($\Delta \to 0$), of the integrated variance covariance matrix.

It is also known that

$$\frac{\sqrt{h/\Delta}\left(\hat{\Omega}_{(ii),t,t+h} - \Omega_{(ii),t,t+h}\right)}{\sqrt{2\int_0^h \Omega^2_{(ii),t,t+\tau} d\tau}} \xrightarrow{d} N(0,1).$$

Since

$$\frac{h/\Delta}{3} \sum_{j=1,\ldots,[h/\Delta]} r^4_{i,t+j\cdot\Delta,\Delta} \xrightarrow{p} \int_0^h \Omega^2_{(ii),t,t+\tau} d\tau,$$

as $\Delta \to 0$, it follows the feasible limiting distribution

$$\frac{\left(\hat{\Omega}_{(ii),t,t+h} - \Omega_{(ii),t,t+h}\right)}{\sqrt{\frac{2}{3}\sum_{j=1,\ldots,[h/\Delta]} r^4_{i,t+j\cdot\Delta,\Delta}}} \xrightarrow{d} N(0,1),$$

showing that the realized variance estimator may be more noisy in high volatility periods.

An additional interesting result concerns the log realized variance, i.e.

$$\frac{\sqrt{h/\Delta}\left(\ln \hat{\Omega}_{(ii),t,t+h} - \ln \Omega_{(ii),t,t+h}\right)}{\sqrt{\frac{2}{3}\frac{\sum_{j=1,\ldots,[h/\Delta]} r^4_{(ii),t+j\cdot\Delta,\Delta}}{\left(\sum_{j=1,\ldots,[h/\Delta]} r^2_{(ii),t+j\cdot\Delta,\Delta}\right)^2}}} \xrightarrow{d} N(0,1),$$

which, according to Barndorff-Nielsen and Shephard (2004), provides a more accurate asymptotic approximation.

On the basis of the above results, daily realized variance ($\hat{\Omega}_{(ii),t}$) can be computed from high frequency observations as follows

$$\hat{\Omega}_{(ii),t} = \sum_{j=1}^{H} r^2_{i,j,t},$$

where $r_{i,j,t}$ is the return on asset i at the intra-daily observation j for day t.[7]

[7] Asymptotic results for the distribution of realized correlations and covariances are also available, but are not discussed since they are not of interest for the paper.

4 Empirical results

4.1 Descriptive statistics

Descriptive statistics for the daily standard deviation and log standard deviation processes are reported in Table 1. As shown in the Table, mean daily volatility is highest at the very short end of the yield curve (overnight rate and one-week rate) and at the nine-month level, ranging between 2.6% and 3.4%, while the other maturities show a lower but similar mean volatility level (about 1.9%). The volatility of volatility is also highest at the very short end of the yield curve (overnight rate, one- and two-week rates), but no clear-cut relationship can be established between the mean level of volatility and the volatility of volatility. According to both the Bera-Jarque and the Kolmogorov-Smirnov tests, deviations from the normality assumption for the log standard deviation processes can be detected for all the maturities, suggesting that the rejection of normality may not be determined only by the violation of the i.i.d hypothesis. The persistence properties of the volatility series, in terms of structural breaks and long memory, are assessed below.

4.2 Structural break tests

As a first step, in order to avoid biased estimation of the persistence parameter[8], as well as that (relative) high frequency cyclical effects may affect the detection of the low frequency deterministic dynamics associated with structural change, the effects of (i) known institutional features of the operational framework, such as the beginning and end of the maintenance period, the number of days between the last allotment and the end of the maintenance period, the first and last trading date of the maintenance period, and (ii) of monetary policy related features, such as press conferences following the Governing Council meeting and interest rate (minimum bid rate) changes, have been removed from the data by regressing the log realized standard deviation series on impulse dummy variables capturing the above mentioned effects.[9] Then, structural break and cobreaking tests have been carried out on the purged series. Two methods for candidate break process estimation have been employed, i.e. the flexible least squares approach of Morana (2006) and the adaptive semiparametric approach of Enders and Lee (2004), as described in Appendix A. Candidate break processes have been computed by

[8] See Cassola and Morana (in press).

[9] See Cassola and Morana (in press) for details on the construction of the dummy variables and their relation to the institutional features of the operational framework of the Eurosystem.

considering trigonometric expansions up to the eighth order for the adaptive semiparametric approach. On the basis of the smoothing criterion and the Monte Carlo results reported in Appendix B, the optimal order of the trigonometric expansion has been set to four. On the other hand, for the flexible least squares approach one hundred different values for the penalization parameter for intercept's dynamics (i.e., $\mu = 100, 200, ..., 10,000$) have been assumed. Then, on the basis of the smoothing criterion and the Monte Carlo results reported in Appendix B the number of selected components has been set to twenty six (i.e., $\mu = 100, 200, ..., 2600$). Both methods are suited to extract a slowly varying deterministic non linear trend, which, according to the decomposition discussed in the methodological section, bears the interpretation of long-run forecast o permanent component for the series from which it is extracted. The estimated candidate break processes and the processes analyzed are plotted in Figure 1. As is shown in the plot, the estimated break processes are smooth, capturing well the trend dynamics in all the series. The two methods yield similar estimates, although, on the basis of a smoothness criterion, adaptive estimation may be preferred. The degree of commonality among the various processes is also strong for all the series, apart from the overnight rate, in both cases. In fact, the correlation coefficient ranges between a minimum of 0.5 and a maximum of 0.9 for all maturities, apart from the overnight rate. For the latter series the highest correlation coefficient is about 0.3 for the FLS method, and 0.4 for the adaptive method.

To test for spurious break processes, possibly due to neglected long memory, four complementary tests robust to the presence of long memory have been implemented, namely the augmented Engle and Kozicki test (Morana, 2002), the Dolado et al. (2004) test, the Teverovsky and Taqqu (1997) test, and the Sibbertsen and Venetis (2004) test. Moreover, in all the cases, to control for small sample effects, the tests have been carried out using critical values obtained by means of the parametric bootstrap.[10] The final evaluation has then been carried out by means of a test based on the Bonferroni bounds principle. Since four tests have been used, the null of no structural change at the 5% level can be rejected if in the worst case the null is rejected at the 1.25% level. The results of the structural break analysis are reported in Table 2, Panels A and B. As is shown in the Table, the structural break tests point to the rejection of the null of no structural change at the 5% significance level only for the one-week rate for the case of FLS estimation, and for the two-week rate only for the case of adaptive estimation. Yet, the evidence

[10] See Sibbertsen and Venetis (2004) and Poskitt (2005) for a justification of the approach.

is not compelling since, according to the Teverovsky and Taqqu (1997) test, the linear relationship between the log variance and the log level of aggregation of the series becomes weaker (the R^2 decreases) in both cases once the break process is removed from the series (from 0.98 to 0.95 and from 0.97 to 0.92, for the one-week rate and the two-week rate, respectively), as for all the other series for which the null of no break process has not been rejected. Moreover, the latter result is not spurious and determined by the approach followed in the estimation of the candidate break processes, since, as shown in Panel 2, the single common break process pointed by the Bierens (2000) test, is not any longer detected once the test is carried out on the break-free series. This latter finding suggests that, if any, the break process would have been adequately modelled by the approaches followed. The results of the cobreaking test may then point to the existence of a single stochastic trend driving the eight log standard deviation processes, i.e. to seven cointegration relationships relating the processes investigated.

Hence, the results of the structural change analysis have some interesting implications for the evaluation of the impact of the changes introduced in March 2004 by the European Central Bank (ECB) to its operational framework for the implementation of monetary policy.[11] These changes were introduced with the aim of stabilizing the bidding behavior of banks at the regular tenders of the ECB, and thereby achieving greater stability of money market conditions. In fact, previous to the changes, expectations of key policy rates changes within the reserve maintenance period created incentives for banks to either overbid or underbid their true liquidity needs at the Eurosystem tenders, depending on whether official rates were expected to increase or decrease, respectively. This in turn created excess volatility in money market conditions (both in liquidity and short-term interest rates), which occasionally interfered with the signaling of the monetary policy stance.

Since the ECB changed its key policy rates more than one year after the introduction of the reform (i.e. beyond the sample covered in this study) it is difficult to test the ability of the new framework to cope with expectations of interest rate changes. Yet, some useful insights can be gauged from the structural break analysis carried out. The stabilizing impact of the operational framework should be visible only at the very short end of the

[11] Firstly, the maturity of the regular main refinancing operations was reduced from two-weeks to one-week; secondly, the reserve maintenance periods, which started always on the 24th of each month and ended on the 23rd, were realigned such that they would start after the ECB's Governing Council policy decison meeting, which takes place normally on the first Thursday of the month. Moreover, the ECB rates decided at that meeting (the standing facilities rates and the minimum bid rate of the weekly tenders) would be applied as of the settlement day of the main refinancing operation following the policy meeting.

money market curve (i.e. below the 1-month maturity). In fact, even without expectations of interest rate changes affecting the behavior of market rates, the reduction in the maturity of the regular refinancing, the calendar adjustment in the reserve maintenance period, and the non-overlapping of liquidity provision across maintenance periods, are changes in the details of the operational framework that may matter for the functioning of the money market. Although some changes in the dynamics of the log-volatility processes seem already visible at the very short end of the money market curve, as lower levels in the break processes of the log-volatility of the one- and two-week rates, and the one-month rate may be observed, the estimated break processes are not statistically significant, i.e. none of the maturities seems to have been permanently affected by the change in the operational framework in the period considered. Yet, this finding does not exclude that the change in the operational framework may have had a persistent impact on money market volatility. The declining trend dynamics pointed out by the estimated candidate break processes, in the light of the outcome of the structural break tests, actually indicate the latter to be the case. Hence, it is possible to conclude that, while the change in the operational framework has not had a permanent impact on money market volatility, i.e. it has not affected the unconditional expectation of volatility, it has exercised persistent effects, i.e. it has affected the conditional expectation of volatility, documented by the declining trend behavior shown by the volatility of the shortest maturities about the time the change took place. It is worthwhile noting that the subtle, yet important, distinction between permanent and persistent effects could have not been gauged without the proposed P-P-NP decomposition.

4.3 Long memory properties

The degree of long memory of the series analyzed has been assessed by means of semiparametric and parametric estimators of the fractional differencing parameter, implemented, given the outcome of the structural break analysis, using the actual series. In order to achieve robust conclusions, both parametric and semiparametric estimators have been employed. The semiparametric estimators employed belong to two classes, i.e. log periodogram estimation (GPH, Geweke and Porter-Hudak, 1983), and local Whittle estimation (Kunsch, 1987; Robinson,1995). Recent extensions to these two classes of estimators have been considered, aiming to improve the performance of the estimators in the presence of: i) short memory dynamics, (Andrews and Guggenberger (2000) bias reduced log periodogram estimator, Andrews and Sun (2004) biased reduced local Whittle estimator, Shimotsu and Phillips

(2002) pooled log periodogram estimator, Moulines and Soulier (1999) broad band log periodogram estimator); ii) observational noise (Sun and Phillips (2003) non linear log periodogram estimator); and iii) non stationarity (Shimotsu and Phillips (2004) exact local Whittle estimator). The estimator of Robinson (1998), which has proved to be robust to both bandwidth selection and the presence of nonstationarity in Monte Carlo experiments, has also been employed. Following Taqqu and Teverovsky (1998), in all the cases the final estimates have been obtained as averages over the stable region closest to the zero frequency. On the other hand, parametric estimation of the fractional differencing operator has been performed by means of exact maximum likelihood (Sowell, 1992), modified profile likelihood (An and Bloomfield, 1993) and non linear least squares estimation (Beran, 1995). For robustness, the final estimate of the fractional differencing parameter for each series has been computed as the median of the estimates obtained by means of the various estimators. The results are reported in Table 3, Panels A-B. As shown in Panel A, the median estimates of the fractional differencing operator are in the range 0.22-0.35, with an average value equal to 0.296 (0.032), and the Bonferroni test for the equality of the fractional differencing parameters does not allow to reject the null of equality of the fractional differencing parameters at the 1% significance level in all cases (Panel B). Hence, in the rest of the analysis a common estimate of the fractional differencing parameter equal to the average value obtained from the eight realized log standard deviation processes, i.e. 0.296 (0.032), has been employed. Finally, no evidence of observational noise could be detected on the basis of the non linear log-periodogram estimator, providing evidence in favour of the volatility proxy employed in the study.

4.3.1 Fractional cointegration analysis

The existence of fractional cointegration has been assessed by means of the Robinson and Yajima (2002) fractional cointegrating rank test. In Table 4, Panels A-B, the results of the fractional cointegration analysis are reported. As shown in Panel A, coherent with previous empirical evidence (Cassola and Morana, in press), the test points to six cointegrating vectors at the 1% significance level, with the two implied common long memory factors explaining 100% of total variance at the selected bandwidth (2 ordinates). The cointegrating vectors have been estimated by means of the frequency domain principal components approach proposed by Morana (2004a, 2005; see also Beltratti and Morana, 2006), applied to the series in levels. In Table 4, Panel A, the unidentified cointegrating vectors are reported. As shown in the Table, the degree of cointegration is moderate, since the error correction residuals still show some persistence. In fact, the estimated fractional differencing parameters range between 0.15 and 0.24, with an average value equal to 0.191 (0.029). Moreover, the Bonferroni test does not allow to reject the null of equality for the estimated fractional differencing parameters at the 5% level. The identification of the cointegration space has been carried out by relying on unitary zero frequency square coherence tests and fractional cointegrating rank tests carried out on subsets of variables. Since the information set is composed of eight variables and there are six cointegrating vectors, the number of identifying restrictions which needs to be imposed on the cointegrating matrix for exact identification, in addition to the normalization restrictions, are thirty. As shown in Table 4, Panel B, on the basis of the fractional cointegrating rank test, carried out on all the possible subsets of variables, five bivariate and one trivariate cointegrating relationships could be identified. The fractional cointegrating rank test points to cointegration for four out of six cases at the 1% significance level, while for the remaining two cases the significance level is just over the 10% level. Yet, since in all cases the implied common fractional trend(s) explains over 97% of total variance and the selection of the threshold level (0.1) for the Robinson and Yajima (2002) test is arbitrary, it is possible to conclude in favour of the selected structure for the cointegration space. Moreover, support for the identified cointegration space is also provided by the zero frequency square coherence analysis. As is shown in Panel B, for all the identified cointegrating vectors the point estimate of the square coherence at the zero frequency is very close to the predicted unitary value, the null of unity is largely non rejected, and the null of zero square coherence is always rejected.[12] Finally, from the identified cointegrating vectors it is possible to note the forward transmission of volatility shocks from the overnight rate to the nine-month rate, moving from all the intermediate maturities considered, i.e. the two-week, one-month, three-month, and the six-month rate. Forward transmission of volatility shocks is also revealed by the sixth cointegrating vector, from the six and nine-month rates to the one-year rate. Interestingly, in only two cases the null of homogeneity is not rejected by the data, i.e. for the two-week and one-week rates and for the nine-month and sixth-month rates.

4.3.2 Permanent-persistent-non persistent decomposition

Since no significant break processes have been found in the log realized volatility series, the permanent components have been estimated by the sample unconditional means of the series and the persistent-non persistent decom-

[12] As shown in Morana (2004b), fractional cointegration implies and is implied by unitary squared coherence at the zero frequency for the series involved.

position has been carried out by means of the Fourier approach of Morana (2006b), selecting the optimal trimming frequency as the closest one to the origin at which the decomposition objective is achieved, i.e. a smoothed persistent component characterized by a degree of persistence non statistically different from the one determined by the semiparametric analysis ($d = 0.296$ (0.032)), and a non persistent component characterized by a degree of persistence non statistically different from zero.[13] The common long memory factors have then been computed by applying the principal components approach, detailed in the methodological section, to the estimated persistent components. On the basis of the results of the cointegration analysis, it is expected that two common long memory factors explain the bulk of persistent fluctuations.

In Figures 3-4 and Table 5 the results of the persistent-non persistent decomposition are reported. As shown in the Table, in all the cases the decomposition has been successful in extracting the long memory component from the series, since the estimated fractional differencing operators for the log volatility series fall in the range 0.28-0.35, and in none of the cases they are statistically different, at the 5% level, from the estimated common value, i.e. 0.296. Moreover, coherent with the results of the cointegration analysis, the first two principal components account for about 75% of total variance. The estimated factors, with 95% confidence bounds, are reported in Figure 3, while in Figure 4 the estimated persistent components, computed from the estimated factors and factor loadings, are compared with the actual series.[14] As shown in the plots, the methodology followed in estimation appears to be robust to trimming, although the second factor appears to have been estimated less precisely. Yet, when robustness is assessed on the basis of the estimated persistent components, it is very hard to distinguish point estimates from their 95% significance bounds, since on average the standard error is only about 10% of the estimated process. As shown by the estimated factor loading matrix, the first factor affects positively all series, while the second factor affects the shorter (up to the two-week horizon) and the longer maturities (from the one-month horizon onwards) with different signs, reflecting an excess persistent volatility component in the longer maturities, relatively to the shorter ones. Overall, the results suggest that the ECB has successfully controlled the shortest end of the yield curve. In fact, the excess

[13] See Appendix A.

[14] Standard errors have been computed from the corresponding cross-sectional distribution determined by the sequential trimming followed, with window length determined by the selected smallest (5) and largest (121) optimal ordinates among the volatility processes analyzed.

volatility of the longer end of the money market curve relative to the shorter end (captured by the second factor) points that the former rates react more promptly to this factor than the latter do. Coherent with previous findings of Cassola and Morana (in press), it is likely that the excess volatility factor captures the flow of "news" about economic conditions, to which the shorter end of the curve is not "free" to react because of the design of the operational framework.

4.4 The propagation of money market volatility shocks

The propagation of volatility shocks in the euro area money market has also been assessed by means of impulse response analysis. Coherent with the long memory properties of the data, impulse responses have been computed from a fractionally cointegrated vector error correction (F-VECM) model. Considering the vector of n $I(d)$, $0 < d < 1$, fractionally cointegrated long memory processes (log volatility processes) $\mathbf{y}_t$, the F-VECM representation (Engle and Granger, 1987; Dittmann, 2004) of the series can be written as

$$\mathbf{\Pi}(L)(1-L)^{\hat{d}}(\mathbf{y}_t - \mathbf{m}) = \left[1-(1-L)^{\hat{d}-\hat{b}}\right](1-L)^{\hat{b}}\boldsymbol{\alpha}\mathbf{z}_t + \boldsymbol{\varepsilon}_t \qquad t = 1, ..., T, \tag{1}$$

where $\mathbf{\Pi}(L) = \mathbf{I}_n - \sum_{i=1}^{p} \mathbf{\Pi}_i L^i$ is a polynomial matrix in the lag operator, with all the roots outside the unit circle, describing the short-run dynamics of the system, $\mathbf{m}$ is a $n \times 1$ vector of intercepts, $\hat{d}$ is the estimated common degree of fractional integration of the actual series, $\hat{b}$, $0 \leq \hat{b} < \hat{d}$, is the estimated degree of fractional integration of the r-variate estimated vector disequilibrium processes $\mathbf{z}_t = \hat{\boldsymbol{\beta}}'\mathbf{y}_t$, with the $n \times r$ matrix of cointegrating vectors denoted by $\boldsymbol{\beta}$, and the $n \times r$ matrix of loadings denoted by $\boldsymbol{\alpha}$, and $\boldsymbol{\varepsilon}_t \tilde{} IID(\mathbf{0}, \mathbf{\Sigma})$. Since, as pointed out in the previous section, the estimation of the cointegration space is semiparametric, the F-VECM model has been estimated following the Engle and Granger (1987) two-step approach, in the framework of the thick modelling strategy proposed by Granger and Jeon (2004), leading to median estimates and 95% confidence bounds for the parameters of interest.[15]

Both information criteria (AIC, BIC) and misspecification tests have been employed for the selection of the lag length of the F-VECM model. While the inclusion of a single lag is sufficient to yield serially uncorrelated residuals,

[15] In the current framework the first step of the Engle and Granger approach requires the determination of the order of integration of all series, the estimation of the cointegrating vectors and the determination of their degree of persistence. The second step then concerns the estimation of the short-run parameters.

according to both the AIC and BIC criteria three lags can be selected.[16] Following the thick modelling strategy, the parameters of the VECM model have then been estimated by considering VECM(1), VECM(2), and VECM(3) structures, with 1000 Monte Carlo replications for each case. Median estimates, as well as 5th and 95th percentiles, have been computed.

The estimated loading matrix for the cointegrating vectors is reported in Table 4, Panels B and C. As is shown in the Table, the results are coherent with the results of the cointegration analysis, since error correcting behavior can be found relatively to all the disequilibria. In particular, the one-month rate disequilibrium is corrected by the two-week rate; the two-week rate disequilibrium by the one-week rate; the six-month rate disequilibrium by the overnight rate, the one-week rate and the own process itself; the three-month rate disequilibrium by the one-month rate; the one-year disequilibrium by the one-month rate and the own process itself; and the nine-month rate disequilibrium by the three-month rate, the one-year rate and the own process itself. Moreover, none of the volatility series is weakly exogenous relatively to the long-run parameters at the 5% significance level.

4.4.1 Identification of structural shocks and impulse response analysis

Exact identification of the structural shocks has been achieved by means of the Choleski decomposition approach. In particular, the following structure has been assumed for the contemporaneous impact matrix

$$\begin{bmatrix} i_{0,t} \\ i_{1w,t} \\ i_{2w,t} \\ i_{1m,t} \\ i_{3m,t} \\ i_{6m,t} \\ i_{9m,t} \\ i_{1y,t} \end{bmatrix} = \begin{bmatrix} * & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \end{bmatrix} \begin{bmatrix} \nu_{i_0,t} \\ \nu_{i_{1w},t} \\ \nu_{i_{2w},t} \\ \nu_{i_{1m},t} \\ \nu_{i_{3m},t} \\ \nu_{i_{6m},t} \\ \nu_{i_{9m},t} \\ \nu_{i_{1y},t} \end{bmatrix}.$$

The ordering of the variables follows standard theoretical assumptions concerning the propagation of shocks along the term structure, with longer maturities showing stronger reactivity to shocks than shorter maturities. The results of the forecast error variance decomposition are reported in Table 7, while median impulse response functions, with 95% confidence bounds, are

[16] Only for the overnight rate the evidence still points to serially correlated residuals, independently of the inclusion of lagged values of the variables.

plotted in Figures 4-11.[17] As shown in Table 7, the own shocks explain the bulk of variability (between 71% and 96%) at all horizons for all variables. Moreover, the evidence points to limited forward propagation of volatility shocks, since for all maturities, apart from the overnight rate and the three-month rate, the own shocks affect non negligibly only the consecutive maturities and, at most, within a time horizon of three months. Furthermore, in none of the cases there is evidence of significant backward propagation of shocks. Finally, according to the forecast error variance decomposition, the bulk of the adjustment is completed within twenty days, with only non significant changes taking place thereafter. The forward propagation of volatility shocks can also be noted from the impulse response functions. The following findings are noteworthy. Firstly, an overnight rate shock affects its own volatility mostly within one day, its effects having faded away already within five days. The overnight rate shock also contemporaneously affects the volatility of other maturities within three months, but the effects of the shock are already not significant (at the 5% level) after one day. For maturities beyond three months the effects are not significant even contemporaneously. Hence, the findings point that liquidity shocks are not transmitted forward along the term structure. On the other hand, the effects of the one-week, two-week and one-month volatility shocks are similar, with shocks propagating forward along the term structure and declining contemporaneous impact as the maturity increases. While the effects of the one-week rate and two-week rate volatility shocks tend to fade away within ten days, for the one-month rate the effects tend to be more long lasting. Interestingly, the effects of the two-week rate volatility shock tend to be weaker than for the other two maturities. Moreover, for all the three maturities significant backward propagation of shocks can be found, with impact decreasing in significance and magnitude as the shortest maturity is approached. Yet, in the light of the results of the forecast error variance decomposition, backward propagation of shocks does not seem to be an important feature for the data analyzed. In the light of the common factor analysis, the persistent factors may be related to the one-week, two-week and one-month rate volatility shocks. In particular, the first persistent factor may be related to the one-week and two-week rate shocks, reflecting the change in the maturity of one of the key ECB policy rates (minimum bid rate of the main refinancing operations) occurred in March 2004, and pointing to forward transmission of persistent shocks, potentially related to ECB monetary policy decisions.

[17] Median impulse responses have been computed by truncating the infinite order VAR representation for the F-VECM model at 25 lags. The median estimates as well as the 95% confidence intervals have been obtained following the thick modelling strategy.

On the other hand, the second persistent factor may be related to the one-month rate volatility shock, possibly reflecting the reaction of the market to macroeconomic news. Finally, also the three-month, six-month, nine-month and one-year rate volatility shocks, show similar properties, with only forward propagation being statistically significant, and declining impact as the maturity increases.

Overall, the results confirm previous findings that, differently from persistent shocks, liquidity shocks are not transmitted along the yield curve. Moreover, the absence of backward transmission of shocks from the longer end of the curve suggests that the ECB controls very closely the shortest end of the yield curve.

5 Conclusions

In the paper the persistence of money market interest rates volatility and the existence of long-run linkages among volatility processes along the term structure has been analyzed. The main findings of the study are as follows. Firstly, the evidence points to a common degree of long memory in the money market interest rates volatility processes. Secondly, evidence of fractional cointegration relationships relating all the series, apart from the overnight rate, and of two common long memory factors, driving the temporal evolution of the volatility processes, has been found. One factor points to forward transmission of persistent volatility shocks along the term structure (first factor), and the other factor points to excess persistent volatility affecting the longer maturities relative to the shorter (second factor). Forward propagation of volatility shocks is also pointed out by the impulse response analysis and the forecast error variance decomposition, showing effects fading away quickly and mostly affecting closest maturities. Interestingly, while the first persistent factor may be related to the ECB policy rate volatility shocks, the second factor may be related to one-month rate volatility shocks. It is conjectured that the second factor in volatility captures the flow of "news" about economic conditions to which the shorter end of the curve is not "free" to react because of the design of the operational framework.

Overall the results suggest that liquidity shocks are not transmitted along the term structure, and that the ECB has successfully controlled the shortest end of the yield curve. In fact, the excess volatility of the longer end of the money market curve relative to the shorter end (captured by the second factor) suggests that the former rates react more promptly to this factor than the latter do. Moreover, negligible evidence of backward transmission of volatility shocks has been found.

6 Appendix A: Estimation of the permanent, persistent, and non persistent components

6.1 Estimation of the permanent component

6.1.1 Flexible least squares estimation

Following Morana (2006), estimation of the break processes has been carried out by means of the FLS estimator. By defining the measurement equation as

$$y_t = \alpha_t + \rho y_{t-1} + \varepsilon_t,$$

and the associated transition equation as

$$\boldsymbol{\gamma}_{i,t} - \boldsymbol{\gamma}_{i,t-1} \simeq 0,$$

where $\boldsymbol{\gamma}_{i,t} = \left[\begin{array}{cc} \alpha_t & \beta \end{array} \right]'$ is the vector parameters in the measurement equation, the FLS problem can be stated as

$$\min_{\boldsymbol{\gamma}} c_{i,M}^2 (\boldsymbol{\gamma}) + c_{i,D}^2 (\boldsymbol{\gamma}),$$

where $c_M^2 (\boldsymbol{\gamma}) = \sum_{t=1}^{T} \varepsilon_t^2$ is the measurement cost,

$$c_D^2 (\boldsymbol{\gamma}) = \sum_{t=2}^{T} [\boldsymbol{\gamma}_t - \boldsymbol{\gamma}_{t-1}]' \boldsymbol{\Upsilon} [\boldsymbol{\gamma}_t - \boldsymbol{\gamma}_{t-1}]$$

is the dynamic cost, and $\boldsymbol{\Upsilon} = \left[\begin{array}{cc} \mu_1 & 0 \\ 0 & \mu_2 \end{array} \right]$ is the matrix containing the penalization terms for parameters dynamics. It is known that $\lim_{\mu \to \infty} \gamma_{j,t,FLS} = \gamma_{j,OLS}$.

The flexible least squares (FLS) estimator of the break process is then denoted as

$$\hat{b}_t(\boldsymbol{\mu}) = \frac{\hat{\alpha}_t}{1 - \hat{\rho}}.$$

A key issue in FLS estimation is the selection of the value of the penalization parameters μ_j. This latter problem has been solved by following a thick modelling strategy (Granger and Jeon, 2004), i.e. by carrying out estimation over a grid of values, i.e. $\mu_1 = \{100, 200, ..., 10, 000\}$, $\mu_2 = 10^6$, and then averaging over a given number of break process candidates, i.e.

$$\hat{b}_{H,t}^* = \frac{1}{H} \sum_{j=1}^{H} \hat{b}_{j,t},$$

where H is determined on the basis of a smoothing criterion, i.e. H is such that

$$\frac{V[\hat{b}^*_{H,t}] - V[\hat{b}^*_{H-1,t}]}{V[\hat{b}^*_{H-1,t}]} = s \simeq 0.$$

In the empirical implementation s can be optimally set to a small value, i.e. 0.01 or 0.005, determined on the basis of the Monte Carlo results reported in the Appendix.

6.1.2 Adaptive semiparametric estimation

Following Enders and Lee (2004), the break process can be modelled by means of the Gallant (1984) flexible functional form

$$b_t = b_0 + b_1 t + \sum_{k=1}^{p} \left(b_{s,k} \sin(2\pi kt/T) + b_{c,k} \cos(2\pi kt/T) \right).$$

Once the number of trigonometric terms is selected, the break process can then be estimated by running the following OLS regression

$$y_t = b_t + \varepsilon_t.$$

Both a smoothing criterion and the Monte Carlo results reported in the Appendix may be followed to determine the order of the expansion k, which has been set to four in the paper. In fact, as far as the Monte Carlo results are concerned, an expansion of the fourth order is optimal in terms of both the Theil inequality coefficient and the root mean square forecast error criteria, being both values negligible. Moreover, in order to assess the impact of increasing the order of the trigonometric expansion on the smoothness of the estimated break process, the break process can been computed for different orders of the expansion, i.e. $k = \{1, 2, ..., 8\}$, and the optimal order of the expansion can be determined on the basis of a smoothing criterion, i.e. k^* is such that

$$\frac{V[\hat{b}_{k^*,t}] - V[\hat{b}_{k^*-1,t}]}{V[\hat{b}_{k^*-1,t}]} = s \simeq 0.$$

6.2 Estimation of the persistent and non persistent components

Consider the stationary process $\{y_t\}_{t=0}^{T-1}$,

$$y_t = s_t + n_t,$$

where s_t is the persistent component ($I(d)$, $d < 0.5$) and n_t is the non persistent component ($I(0)$).

Then, following Morana (2006b), estimation of the persistent component s_t, can be achieved as follows. Firstly, the discrete Fourier transform of y_t is computed

$$\tilde{y}_t = \frac{1}{N} \sum_{k=0}^{T-1} y_t e^{i2\pi k/T}.$$

Then, the portion of the transformed process corresponding to the non persistent component is discarded by setting

$$\tilde{y}_t^* = \begin{cases} \tilde{y}_t & 0 \leq t \leq H \\ 0 & t > H \end{cases}.$$

Finally, s_t is estimated by applying the inverse discrete Fourier transform to $\tilde{y}_t^*$, yielding

$$s_t = \frac{1}{N} \sum_{k=0}^{T-1} \tilde{y}_t^* e^{-i2\pi k/T}.$$

A two-step procedure for the determination of the trimming frequency $2\pi H/T$ is proposed. Once the degree of fractional integration ($\hat{d}_y$) of the process y_t has been determined, candidate persistent processes are computed by allowing H to vary, i.e. $H = \{3, 4, ...T - 1\}$, computing, in correspondence of each value of H, the degree of persistence of the reconstructed persistent ($\hat{d}_{s,H}$) and non persistent ($\hat{d}_{n,H}$) components. The optimal trimming frequency is then determined by selecting H in such a way that

$$\hat{d}_{s,H} \simeq \hat{d}_y \text{ and } d_{n,H} \simeq 0,$$

i.e. as the frequency at which the reconstructed persistent component has a degree of persistence not statistically different from the one of the actual process and the reconstructed non persistent component has a degree of persistence not statistically different from zero.[18] In the case more than a frequency satisfied this latter criterion, the farthest one from the zero frequency may be selected in order not to disregard signal potentially belonging to the persistent component. On the other hand, if a smoothed version of the unobserved component is sought, or when the non persistent component may still be an integrated process $I(b)$, although of lower order than the actual series, i.e. $b < d$, the optimal trimming frequency may be set equal to the closest one to the zero frequency, still satisfying the above persistence

[18]See Pollock (2005) for additional details on Fourier series based signal extraction methods.

requirements. The Monte Carlo results reported in Appendix B provide full support to the proposed methodology.[19]

7 Appendix B: Break process estimation, Monte Carlo results

The following data generating process has been assumed for the log realized volatility process

$$
\begin{aligned}
(1-L)^d y_t &= b_t + \varepsilon_t \\
& \varepsilon_t \tilde{\ } n.i.d.(0,1) \\
b_{1,t} &= \begin{cases} 0 & 1 \leq t \leq 500 \\ 1 & 501 \leq t \leq 1000 \end{cases} \\
b_{2,t} &= \begin{cases} 0 & 1 \leq t \leq 300 \\ 1 & 301 \leq t \leq 700 \\ -0.5 & 701 \leq t \leq 1000 \end{cases},
\end{aligned}
$$

with $s = \{0.01, 0.007, 0.005, all\}$, $d = \{0.2, 0.3, 0.4\}$, $b_t = \{b_{1,t}, b_{2,t}\}$, $t = 1, ..., 1000$. The number of replications has been set to 500 for each case.

The performance of the estimators has been assessed with reference to the ability of recovering the unobserved components b_t, s_t, and f_t, respectively. The Theil inequality coefficient (IC) and the root mean square forecast error ($RMSFE$) have been employed in the evaluation

$$
\begin{aligned}
RMSFE &= \sqrt{\frac{1}{T}\sum_{t=1}^{T}(x_t^* - x_t)^2} \\
IC &= \frac{RMSFE}{\sqrt{\frac{1}{T}\sum_{t=1}^{T} x_t^{*2}} + \sqrt{\frac{1}{T}\sum_{t=1}^{T} x_t^2}},
\end{aligned}
$$

where $x_t^* = \frac{1}{n}\sum_{j=1}^{n} \hat{x}_{j,t}$ and $\hat{x}_{j,t}$ is the estimated unobserved component at time t for replication j, $x_t = \{b_t, s_t, f_t\}$.

The results of the Monte Carlo exercise for the break process estimation approach are reported in Table 8. As is shown in Table 8, the performance

[19] Note that the proposed methodology could also be employed to compute a signal-noise decomposition.

of both the FLS and the adaptive estimator is not affected by the degree of persistence of the series, increasing with the number of trigonometric components included in the specification (adaptive) and as the smoothing threshold s (FLS) is lowered. While according to the Theil inequality coefficient the adaptive estimator performs well already when a single trigonometric component is included, a substantial improvement may be noticed by adding an additional trigonometric component for the single break case and three additional components for the two breaks case. On the other hand, for the FLS estimator averaging over all the candidate break processes does not seem to be optimal. An optimal threshold can be found by assessing the contribution of the additional candidate to the smoothness of the break process, yielding an optimal smoothing threshold (s) equal to 0.01. Moreover, both the FLS and adaptive estimators perform best in the case of a single break point than for the case of two break points. While both approaches may successfully recover an unobserved break process, the FLS approach seems to perform slightly better than the adaptive one.

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Table 1, Panel A: Summary statistics for daily realized standard deviations

	i_0	i_{1w}	i_{2w}	i_{1m}	i_{3m}	i_{6m}	i_{9m}	i_{1y}
mean	3.296	3.355	1.896	1.879	1.998	1.875	2.554	1.877
s.d.	6.367	9.759	7.368	3.511	2.743	2.707	3.012	2.818
sk	4.578	6.765	13.066	4.045	3.682	6.101	4.097	8.988
ek	29.180	56.511	210.45	21.212	19.260	47.226	26.289	108.73
min	0.104	0.010	0.021	0.015	0.048	0.071	0.228	0.235
max	64.573	109.72	147.70	32.727	26.128	31.156	34.001	42.513
N_{BJ}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 1, Panel B: Summary statistics for daily realized log standard deviations

	i_0	i_{1w}	i_{2w}	i_{1m}	i_{3m}	i_{6m}	i_{9m}	i_{1y}
mean	0.175	-0.041	-0.341	-0.345	0.119	0.232	0.545	0.316
s.d.	1.352	1.392	1.173	1.348	1.035	0.808	0.831	0.668
sk	0.582	0.661	0.572	0.369	0.303	0.535	0.563	1.215
ek	-0.475	0.629	1.369	-0.197	-0.119	1.222	0.000	2.801
min	-2.262	-4.630	-3.882	-4.215	-3.044	-2.651	-1.479	-1.450
max	4.168	4.698	4.995	3.488	3.263	3.439	3.526	3.750
N_{BJ}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
KS	2.849	2.241	1.697	2.074	1.66	1.509	2.491	2.857

The table reports summary statistics for daily realized standard deviations (Panel A) and log realized standard deviations (Panel B). i_j , $j = 0, 1w, 2w, 1m, 3m, 6m, 9m, 1y$, denotes the overnight rate, the one-week rate, the two-week rate, the one-month rate, the three-month rate, the six month rate, the nine-month rate and the one-year rate, respectively. N_{BJ} is the p-value of the Bera-Jarque normality test, while KS is the value of the Kolmogorov-Smirnov test (the 99% and 95% critical values are 1.63 and 1.36, respectively). The time span analysed is 28/11/2000:22/04/05, for a total of 952 daily observations.

Table 2, Panel A: Structural break and cobreaking tests, FLS estimation

	i_0	i_{1w}	i_{2w}	i_{1m}	i_{3m}	i_{6m}	i_{9m}	i_{1y}
SBT	0.99	0.01	0.27	0.11	0.05	0.05	0.19	0.24
R^2	0.98	0.98	0.97	0.97	0.98	0.95	0.97	0.97
R^2_{bf}	0.95	0.95	0.93	0.91	0.94	0.93	0.94	0.95

	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$	$r=6$	$r=7$	$r=8$
CBT	0.019	0.024	0.030	0.064	0.089	0.214	0.627	2.541^*
CBT_{bf}	0.014	0.016	0.019	0.020	0.025	0.030	0.031	0.034
95%	0.466	0.674	0.860	1.035	1.219	1.411	1.598	1.785

Table 2, Panel B: Structural break and cobreaking tests, adaptive estimation

	i_0	i_{1w}	i_{2w}	i_{1m}	i_{3m}	i_{6m}	i_{9m}	i_{1y}
SBT	0.27	0.04	0.01	0.06	0.11	0.04	0.15	0.15
R^2	0.98	0.98	0.97	0.97	0.98	0.95	0.97	0.97
R^2_{bf}	0.95	0.95	0.92	0.90	0.93	0.93	0.94	0.95

	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$	$r=6$	$r=7$	$r=8$
CBT	0.019	0.024	0.030	0.064	0.089	0.214	0.627	2.541^*
CBT_{bf}	0.011	0.011	0.014	0.015	0.016	0.018	0.021	0.0254
95%	0.466	0.674	0.860	1.035	1.219	1.411	1.598	1.785

The table reports the results of the (Bonferroni bounds) structural break test (SBT), and the R^2 of the log-log variance regression obtained from the Teverovsky and Taqqu (1997) test (actual series and break-free (bf) series). The table also reports the results of the Bierens (2000) cobreaking test (CBT) for the null of up to r cobreaking vectors for the actual and break-free (bf) series. "*" denotes rejection at the 5% significance level; "95%" denotes the critical value for the test.

Table 3, Panel A: Median estimated fractional differencing operators

	i_0	i_{1w}	i_{2w}	i_{1m}	i_{3m}	i_{6m}	i_{9m}	i_{1y}
d	0.217	0.268	0.333	0.352	0.332	0.279	0.292	0.292
	0.032	0.031	0.030	0.029	0.031	0.035	0.032	0.032

Table 3, Panel B: Test for the equality of the fractional differencing operators

	i_0	i_{1w}	i_{2w}	i_{1m}	i_{3m}	i_{6m}	i_{9m}	i_{1y}
i_0		0.178	0.003	$3E-4$	0.002	0.097	0.047	0.047
i_{1w}	0.252		0.091	0.025	0.090	0.768	0.525	0.525
i_{2w}	0.008	0.132		0.618	0.979	0.155	0.286	0.286
i_{1m}	0.002	0.048	0.649		0.593	0.048	0.108	0.108
i_{3m}	0.010	0.144	0.982	0.638		0.154	0.288	0.288
i_{6m}	0.191	0.814	0.241	0.108	0.257		0.727	0.727
i_{9m}	0.097	0.590	0.350	0.165	0.369	0.784		1.000
i_{1y}	0.097	0.590	0.350	0.165	0.369	0.784	1.000	

Panel A reports the estimated fractional differencing operators, with standard error in brackets. Panel B reports the p-values of the tests for the equality of the fractional differencing parameter. The upper triangular matrix refers to the Robinson and Yajima (2002) test; the lower triangular matrix to the standard test for the equality of two estimated parameters.

Table 4, Panel A: Fractional cointegrating rank test, all variables

$b=2$					eig	pv
	1%	5%	10%	TV	1.531	0.898
$r=1$	0.000	0.000	0.000	0.013	0.175	0.102
$r=2$	0.000	0.000	0.000	0.025	0.000	0.000
$r=3$	0.000	0.000	0.000	0.038	0.000	0.000
$r=4$	0.000	0.000	0.000	0.050	0.000	0.000
$r=5$	0.000	0.000	0.000	0.063	0.000	0.000
$r=6$	0.000	0.000	0.000	0.075	0.000	0.000
$r=7$	0.256	0.211	0.187	0.088	0.000	0.000

Table 4, Panel B: Fractional cointegrating rank test, subsets

$b=2$	(i_{2w}, i_{1w})			$b=2$	(i_{1m}, i_{2w})			
eig	0.424	0.002		eig	0.712	0.019		
pv	0.996	0.004		pv	0.973	0.027		
	1%	5%	10%		1%	5%	10%	TV
$r=1$	0.010	0.008	0.007	$r=1$	0.069	0.057	0.050	0.050

$b=2$	(i_{3m}, i_{1m})			$b=2$	(i_{6m}, i_{3m})			
eig	0.647	0.006		eig	0.386	0.002		
pv	0.991	0.009		pv	0.996	0.004		
	1%	5%	10%		1%	5%	10%	TV
$r=1$	0.024	0.019	0.017	$r=1$	0.01	0.008	0.007	0.050

$b=2$	(i_{9m}, i_{6m})				$b=2$	(i_{1y}, i_{2w}, i_0)			
eig	0.438	0.012			eig	0.308	0.082	0.000	
pv	0.973	0.027			pv	0.790	0.210	0.000	
	1%	5%	10%	TV		1%	5%	10%	TV
$r=1$	0.071	0.059	0.052	0.050	$r=1$	0.000	0.000	0.000	0.033
					$r=2$	0.484	0.403	0.361	0.067

Panels A and B report the results of the Robinson and Yajima (2002) fractional cointegrating rank test. *eig* denotes the estimated eigenvalues, *pv* the proportion of explained variance, $rank = i$, $i = 1, ..., 2$, denotes the corresponding test at the given significance level (1%, 5%, 10%), TV is the threshold value for the test, computed as $0.1 * r/n$, and b is the selected bandwidth.

Table 5, Panel A: Estimated (unidentified) cointegrating vectors

	CV_1	CV_2	CV_3	CV_4	CV_5	CV_6
i_0	−0.337	−0.113	0.946	0.144	−0.978	0.527
i_{1w}	−0.240	−1.280	−0.829	0.216	−0.457	−0.214
i_{2w}	−1.245	1.000	−0.017	−0.319	−0.033	−0.640
i_{1m}	1.000	0.354	0.121	−0.178	−1.326	0.014
i_{3m}	0.103	0.832	−0.188	1.000	0.492	0.329
i_{6m}	0.216	−0.750	1.000	0.106	1.054	−0.498
i_{9m}	−1.053	−0.375	0.009	−0.205	0.240	1.000
i_{1y}	0.821	0.277	−0.257	0.561	1.000	0.538
C_0^2	1.000	1.000	1.000	1.000	1.000	1.000
T_0	0.000	0.000	0.000	0.000	0.000	0.000
T_1	0.999	0.999	0.999	0.999	0.999	0.999
b	0.189 (0.029)	0.150 (0.029)	0.149 (0.029)	0.236 (0.029)	0.238 (0.029)	0.185 (0.029)

Table 5, Panel B: Estimated (identified) cointegrating vectors

	CV_1	CV_2	CV_3	CV_4	CV_5	CV_6
i_0	0	0	0	0	−0.603 (0.126)	0
i_{1w}	0	−0.992 (0.064)	0	0	0	0
i_{2w}	−1.459 (0.103)	1	0	0	−0.695 (0.163)	0
i_{1m}	1	0	0	−0.496 (0.051)	0	0
i_{3m}	0	0	−1.359 (0.085)	1	0	0
i_{6m}	0	0	1	0	0	−0.833 (0.083)
i_{9m}	0	0	0	0	0	1
i_{1y}	0	0	0	0	1	0
C_0^2	0.875	0.984	0.983	0.945	1.000	0.833
T_0	0.000	0.000	0.000	0.000	0.000	0.000
T_1	0.974	0.996	0.996	0.988	0.999	0.964

Table 5, Panel C: Loading matrix

	i_0	i_{1w}	i_{2w}	i_{1m}
CV_1	0.331 $(-0.03$ - $0.92)$	0.258 $(-0.26$ - $0.74)$	0.458^* $(0.06$ - $0.90)$	0.107 $(-0.40$ - $0.55)$
CV_2	0.079 $(-0.47$ - $0.59)$	0.699^* $(0.23$ - $1.15)$	-0.032 $(-0.50$ - $0.36)$	0.099 $(-0.30$ - $0.55)$
CV_3	-1.082^* $(-2.01$ - $-0.52)$	0.728^* $(0.02$ - $1.62)$	0.141 $(-0.49$ - $0.81)$	-0.077 $(-0.76$ - $0.73)$
CV_4	0.192 $(-0.50$ - $1.06)$	-0.337 $(-1.12$ - $0.33)$	-0.101 $(-0.74$ - $0.43)$	0.688^* $(0.14$ - $1.35)$
CV_5	0.066 $(-0.18$ - $0.41)$	0.017 $(-0.31$ - $0.31)$	-0.040 $(-0.30$ - $0.25)$	0.295^* $(0.01$ - $0.64)$
CV_6	-0.442 $(-1.32$ - $0.30)$	-0.549 $(-1.29$ - $0.18)$	0.131 $(-0.62$ - $0.76)$	-0.340 $(-1.18$ - $0.38)$

Table 5, Panel D: Loading matrix

	i_{3m}	i_{6m}	i_{9m}	i_{1y}
CV_1	0.125 $(-0.20$ - $0.52)$	0.174 $(-0.09$ - $0.46)$	0.251 $(-0.11$ - $0.53)$	0.043 $(-0.23$ - $0.28)$
CV_2	0.047 $(-0.28$ - $0.47)$	-0.020 $(-0.28$ - $0.23)$	-0.228 $(-0.56$ - $0.09)$	-0.137 $(-0.43$ - $0.05)$
CV_3	0.171 $(-0.48$ - $0.74)$	-0.505^* $(-1.01$ - $-0.15)$	0.329 $(-0.12$ - $0.78)$	0.257 $(-0.05$ - $0.77)$
CV_4	0.191 $(-0.48$ - $0.64)$	0.192 $(-0.18$ - $0.67)$	0.294 $(-0.06$ - $0.86)$	0.159 $(-0.14$ - $0.56)$
CV_5	0.079 $(-0.16$ - $0.28)$	-0.101 $(-0.27$ - $0.08)$	0.019 $(-0.17$ - $0.20)$	-0.167^* $(-0.32$ - $-0.02)$
CV_6	-0.678^* $(-1.34$ - $-0.11)$	0.320 $(-0.13$ - $0.73)$	-0.569^* $(-1.01$ - $-0.16)$	-0.173^* $(-0.60$ - $0.21)$

Panel A reports the unidentified estimated cointegrating vectors, Panel B reports the identified estimated cointegrating vectors with Christensen and Nielsen (in press) asymptotic standard error in parenthesis. C_0^2 is the estimated simple or multiple square coherence at the zero frequency; T_0 and T_1 denote the p-values of the zero and unitary squared coherence at the zero frequency test, respectively; b is the estimated fractional differencing parameter for the cointegrating vectors. Panels C and D report the estimated median loadings, with 95% confidence intervals in parenthesis. "*" denotes rejection of the null of zero coefficient at the 5% level.

Table 6: Persistent-Non Persistent decomposition

	d_p	d_{np}	m	f_1	f_2
i_0	0.353	0.020	5	0.127	-0.232
	(0.071)	(0.049)		(0.056)	(0.066)
i_{1w}	0.290	0.000	42	0.618	-0.292
	(0.061)	(0.022)		(0.059)	(0.054)
i_{2w}	0.281	0.084	63	0.646	-0.295
	(0.059)	(0.065)		(0.027)	(0.006)
i_{1m}	0.300	0.014	121	0.965	0.119
	(0.062)	(0.093)		(0.008)	(0.007)
i_{3m}	0.282	0.076	73	0.533	0.066
	(0.057)	(0.090)		(0.062)	(0.031)
i_{6m}	0.316	0.066	64	0.402	0.284
	(0.067)	(0.065)		(0.016)	(0.034)
i_{9m}	0.290	0.083	83	0.317	0.369
	(0.062)	(0.062)		(0.036)	(0.032)
i_{1y}	0.287	0.069	90	0.246	0.078
	(0.061)	(0.091)		(0.028)	(0.026)
pv				0.635	0.126
				(0.029)	(0.007)

The table reports the estimated fractional differencing operator for the estimated persistent (p) and non-persistent (np) components, the optimal trimming ordinate (m), and the estimated factor loading matrix (f_1, f_2), with the proportion of total variance (pv) explained by each factor. Standard errors are reported in brackets.

Table 7: Forecast error variance decomposition

1- day	ν_{i_0}	$\nu_{i_{1w}}$	$\nu_{i_{2w}}$	$\nu_{i_{1m}}$	$\nu_{i_{3m}}$	$\nu_{i_{6m}}$	$\nu_{i_{9m}}$	$\nu_{i_{1y}}$
i_0	96.58	1.11	0.78	0.06	0.49	0.17	0.30	0.05
i_{1w}	3.05	94.09	0.02	0.28	0.04	0.07	0.05	0.05
i_{2w}	3.02	19.51	76.32	0.55	0.05	0.14	0.04	0.06
i_{1m}	1.64	13.59	7.21	76.26	0.12	0.33	0.05	0.07
i_{3m}	1.22	2.71	3.11	8.69	83.55	0.04	0.14	0.07
i_{6m}	0.15	3.42	0.65	4.18	3.18	86.34	1.28	0.26
i_{9m}	0.14	1.75	0.23	3.26	1.87	14.01	78.40	0.04
i_{1y}	0.25	2.58	0.69	3.55	0.61	6.22	10.73	74.77
5- day	ν_{i_0}	$\nu_{i_{1w}}$	$\nu_{i_{2w}}$	$\nu_{i_{1m}}$	$\nu_{i_{3m}}$	$\nu_{i_{6m}}$	$\nu_{i_{9m}}$	$\nu_{i_{1y}}$
i_0	89.42	2.87	2.50	1.05	1.08	0.78	0.82	0.21
i_{1w}	3.19	89.60	3.73	1.33	0.35	0.27	0.57	0.20
i_{2w}	3.25	19.01	73.48	2.06	0.26	0.45	0.26	0.23
i_{1m}	1.85	13.60	7.23	74.17	0.94	0.63	0.48	0.25
i_{3m}	1.60	2.91	3.39	8.94	80.90	0.42	0.69	0.31
i_{6m}	0.38	4.22	0.91	5.75	3.62	82.53	1.71	0.41
i_{9m}	0.34	2.12	0.63	4.10	2.03	15.32	74.83	0.20
i_{1y}	1.11	3.31	0.91	4.11	0.79	6.20	10.34	72.80
10- day	ν_{i_0}	$\nu_{i_{1w}}$	$\nu_{i_{2w}}$	$\nu_{i_{1m}}$	$\nu_{i_{3m}}$	$\nu_{i_{6m}}$	$\nu_{i_{9m}}$	$\nu_{i_{1y}}$
i_0	88.40	3.10	2.59	1.10	1.20	1.01	1.07	0.25
i_{1w}	3.22	88.56	3.99	1.69	0.45	0.31	0.67	0.23
i_{2w}	3.28	18.95	72.84	2.44	0.35	0.51	0.32	0.27
i_{1m}	2.02	13.42	7.12	73.32	1.49	0.71	0.60	0.28
i_{3m}	1.68	2.90	3.55	8.84	80.34	0.46	0.85	0.37
i_{6m}	0.43	4.51	0.98	6.05	4.02	81.26	1.71	0.46
i_{9m}	0.42	2.21	0.76	4.27	2.15	15.80	73.68	0.26
i_{1y}	1.37	3.58	0.98	4.54	0.85	6.24	10.09	71.83
20- day	ν_{i_0}	$\nu_{i_{1w}}$	$\nu_{i_{2w}}$	$\nu_{i_{1m}}$	$\nu_{i_{3m}}$	$\nu_{i_{6m}}$	$\nu_{i_{9m}}$	$\nu_{i_{1y}}$
i_0	87.61	3.21	2.76	1.13	1.28	1.18	1.26	0.27
i_{1w}	3.21	87.73	4.15	1.90	0.53	0.35	0.78	0.26
i_{2w}	3.36	18.76	72.36	2.76	0.45	0.54	0.40	0.29
i_{1m}	2.14	13.12	7.02	72.48	2.15	0.74	0.72	0.32
i_{3m}	1.78	2.91	3.70	8.74	79.56	0.50	1,04	0.41
i_{6m}	0.49	4.71	1.03	6.33	4.38	79.90	1.73	0.49
i_{9m}	0.47	2.29	0.86	4.49	2.25	16.10	72.61	0.32
i_{1y}	1.58	3.85	1.06	4.88	0.91	6.25	9.97	70.86

The Table reports the median forecast error variance decomposition.

Table 8: Break process estimation, Monte Carlo results

$1\ break\ point : IC$

FLS				$ADAPTIVE$			
$s \backslash d$	0.2	0.3	0.4	$c \backslash d$	0.2	0.3	0.4
0.01	0.04	0.04	0.05	1	0.13	0.13	0.13
0.007	0.04	0.04	0.05	2	0.10	0.10	0.10
0.005	0.05	0.05	0.05	3	0.09	0.09	0.09
all	0.07	0.07	0.07	4	0.08	0.08	0.08

$1\ break\ point :\ RMSFE$

FLS				$ADAPTIVE$			
$s \backslash d$	0.2	0.3	0.4	$c \backslash d$	0.2	0.3	0.4
0.01	0.06	0.06	0.07	1	0.17	0.17	0.17
0.007	0.06	0.06	0.07	2	0.14	0.14	0.14
0.005	0.06	0.07	0.07	3	0.12	0.12	0.12
all	0.10	0.10	0.10	4	0.11	0.11	0.11

$2\ break\ points : IC$

FLS				$ADAPTIVE$			
$s \backslash d$	0.2	0.3	0.4	$c \backslash d$	0.2	0.3	0.4
0.01	0.08	0.08	0.08	1	0.23	0.23	0.23
0.007	0.08	0.08	0.09	2	0.19	0.19	0.19
0.005	0.09	0.09	0.10	3	0.17	0.17	0.17
all	0.14	0.14	0.14	4	0.13	0.13	0.13

$2\ break\ points : RMSFE$

FLS				$ADAPTIVE$			
$s \backslash d$	0.2	0.3	0.4	$c \backslash d$	0.2	0.3	0.4
0.01	0.10	0.10	0.11	1	0.31	0.31	0.31
0.007	0.11	0.11	0.12	2	0.25	0.25	0.25
0.005	0.12	0.12	0.13	3	0.23	0.23	0.23
all	0.18	0.18	0.18	4	0.18	0.18	0.18

The Table reports the Theil inequality coefficient (IC) and the root mean square forecast error ($RMSFE$) for the FLS and the adaptive break process estimator.

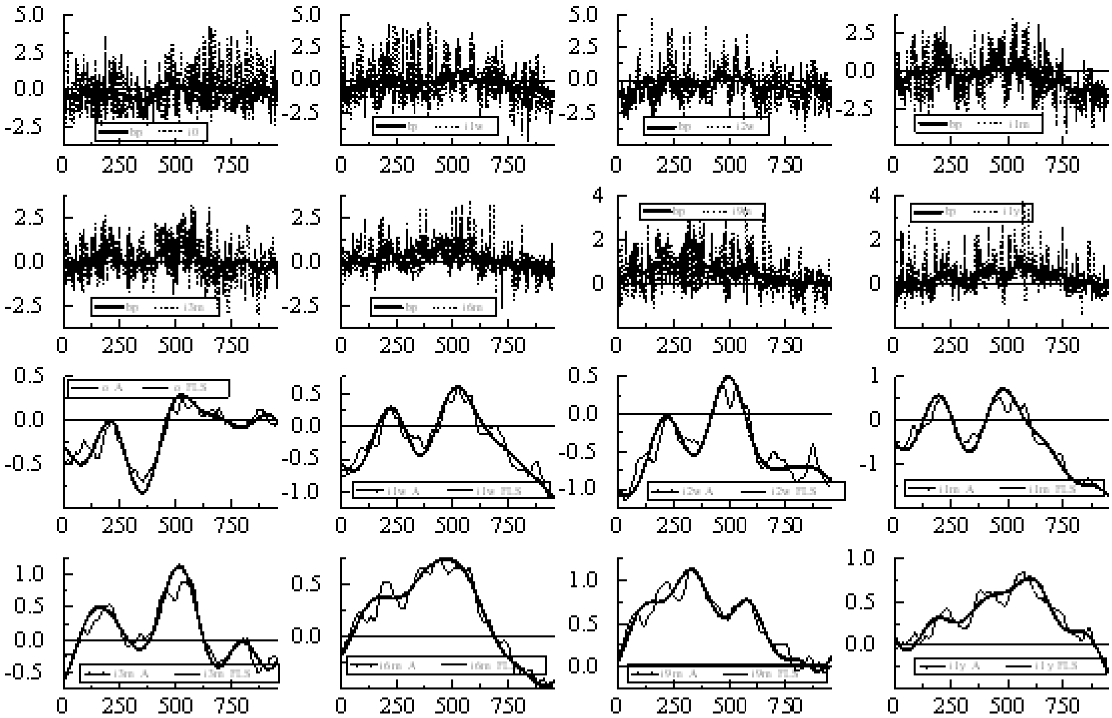

Figure 1: Plots 1-6: log realized standard deviations: actual series and FLS candidate break processes (overnight rate (i_0), one-week rate (i_{1w}), two-week rate (i_{2w}), one-month rate (i_{1m}), three-month rate (i_{3m}), six month rate (i_{6m}), nine month rate (i_{9m}), twelve-month rate (i_{1y})). Plots 7-12 FLS (FLS) and adaptive (A) candidate break processes.

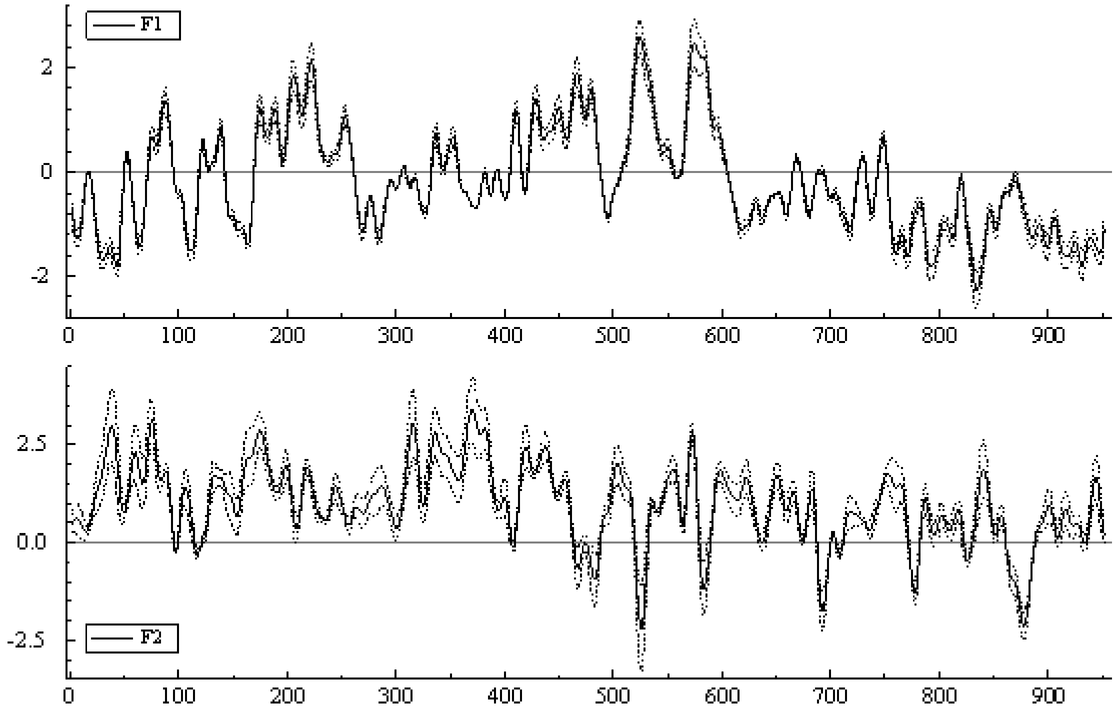

Figure 2: Persistent factors with 95% confidence bounds.

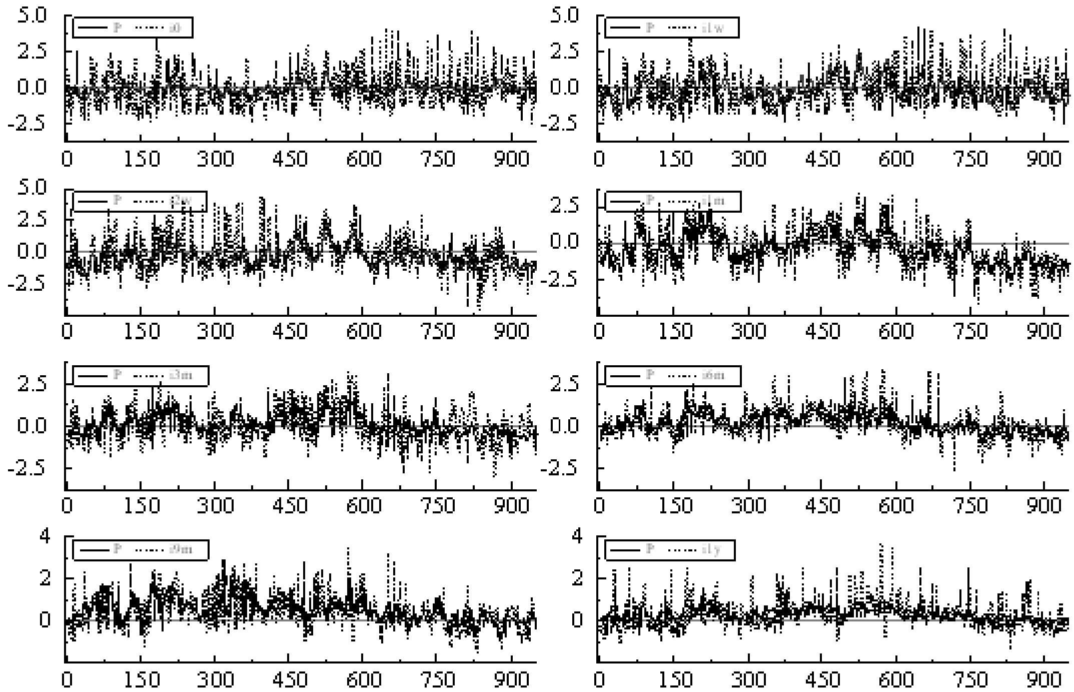

Figure 3: Persistent components (with 95% confidence bounds) and actual series.

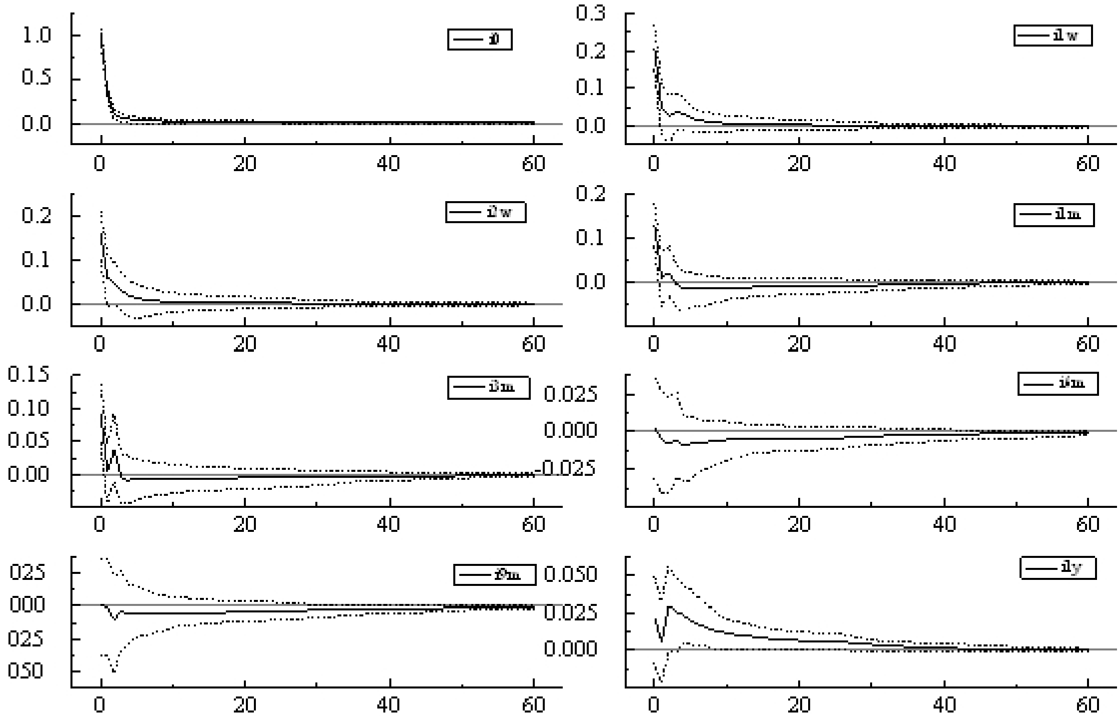

Figure 4: Impulse response functions; unitary overnight rate shock.

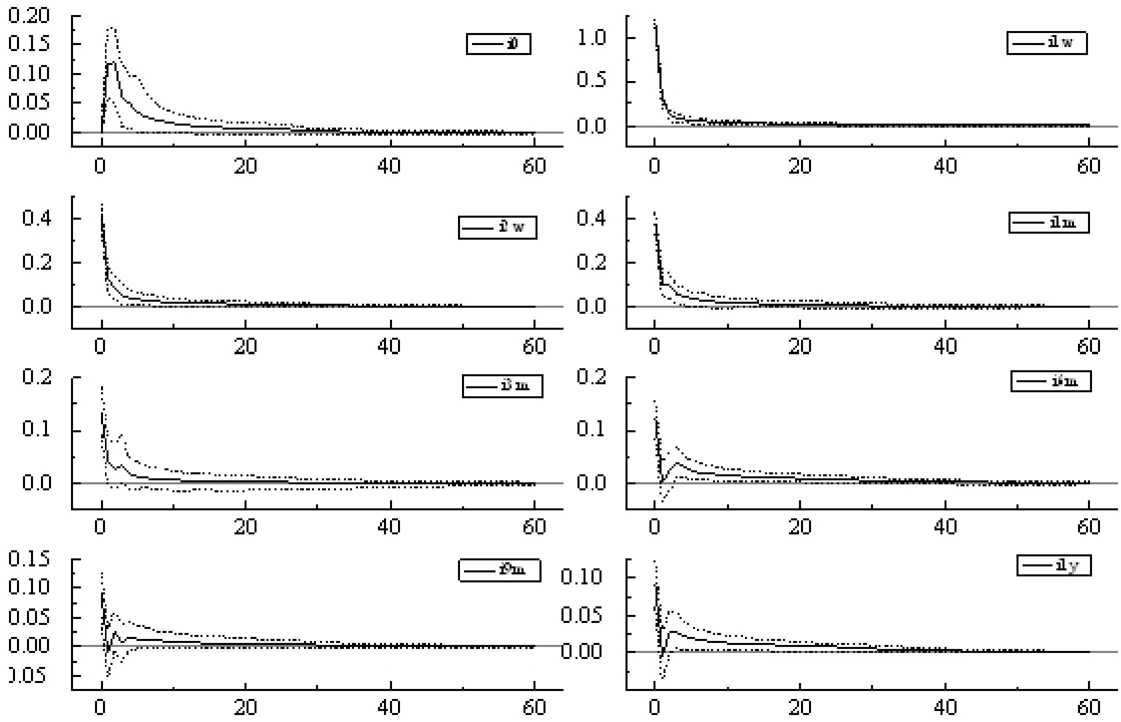

Figure 5: Impulse response functions; unitary one-week rate shock.

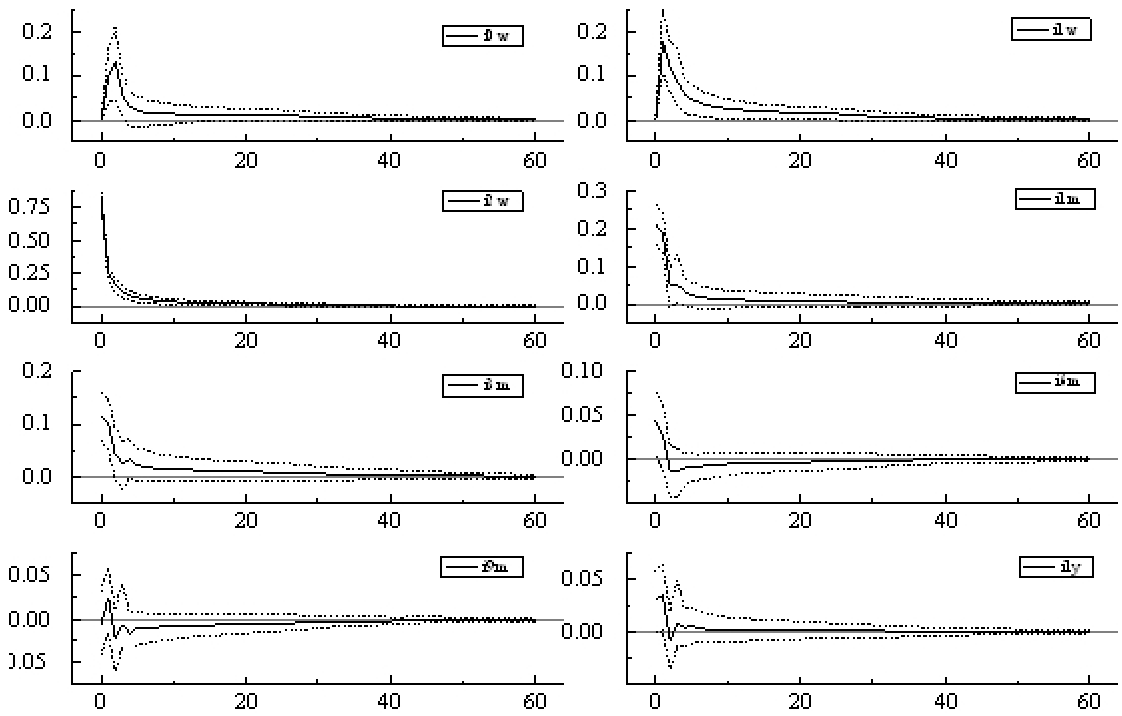

Figure 6: Impulse response functions; unitary two-week rate shock.

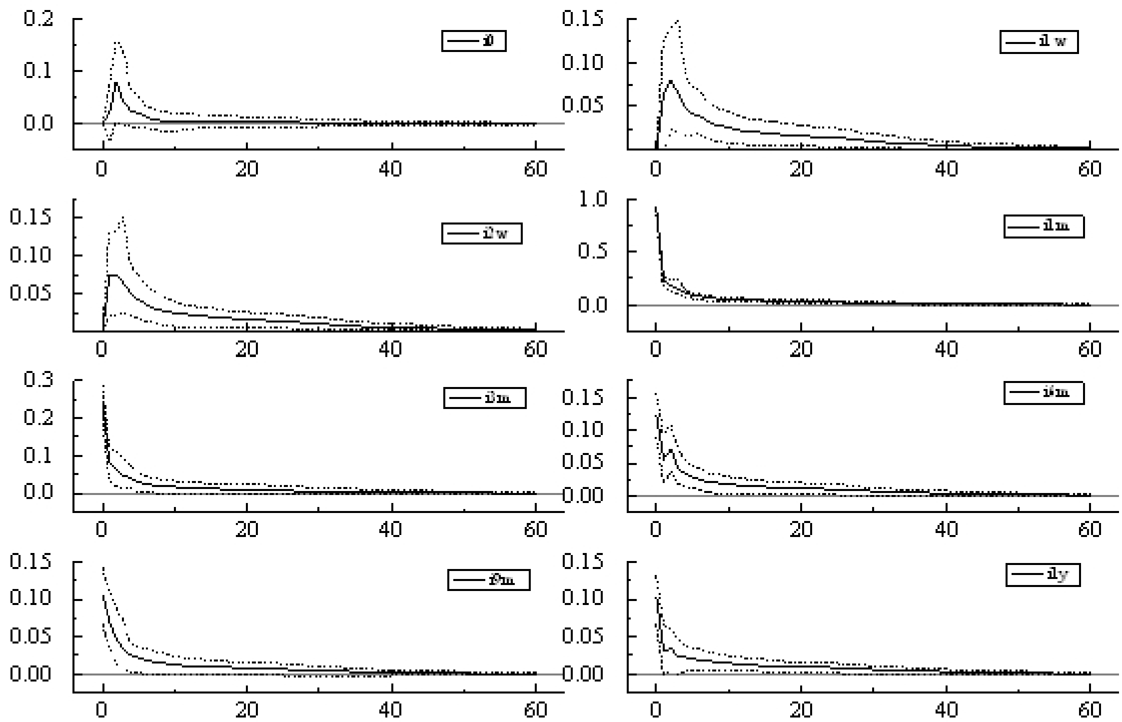

Figure 7: Impulse response functions; unitary one-month rate shock.

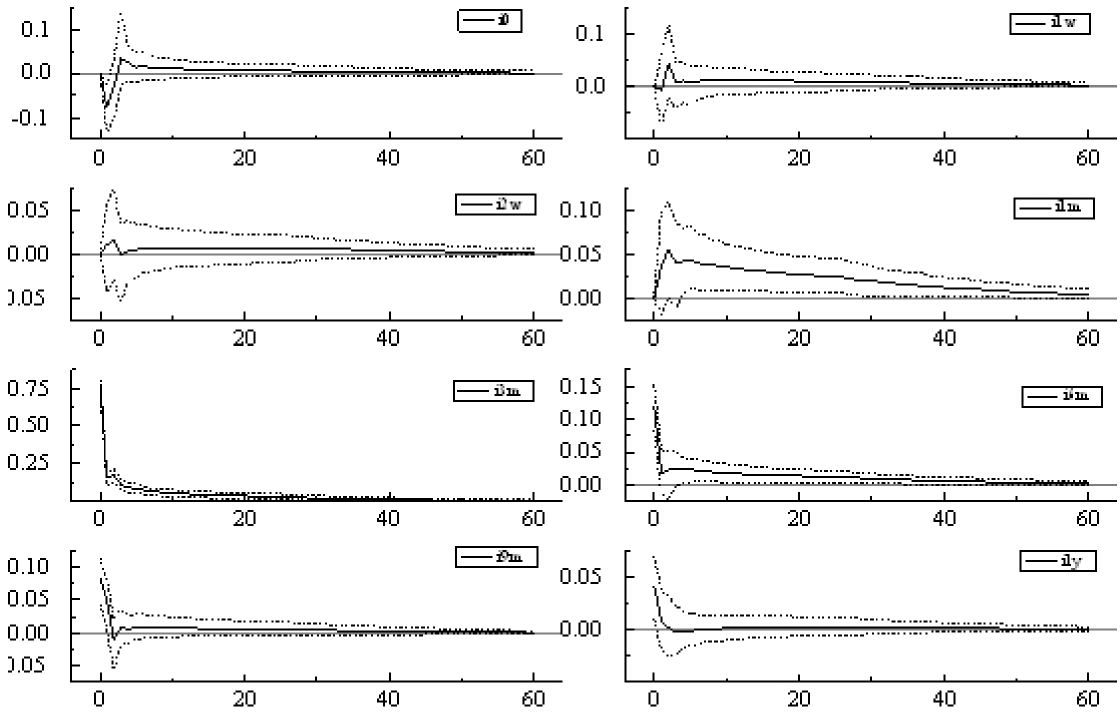

Figure 8: Impulse response functions; unitary three-month rate shock.

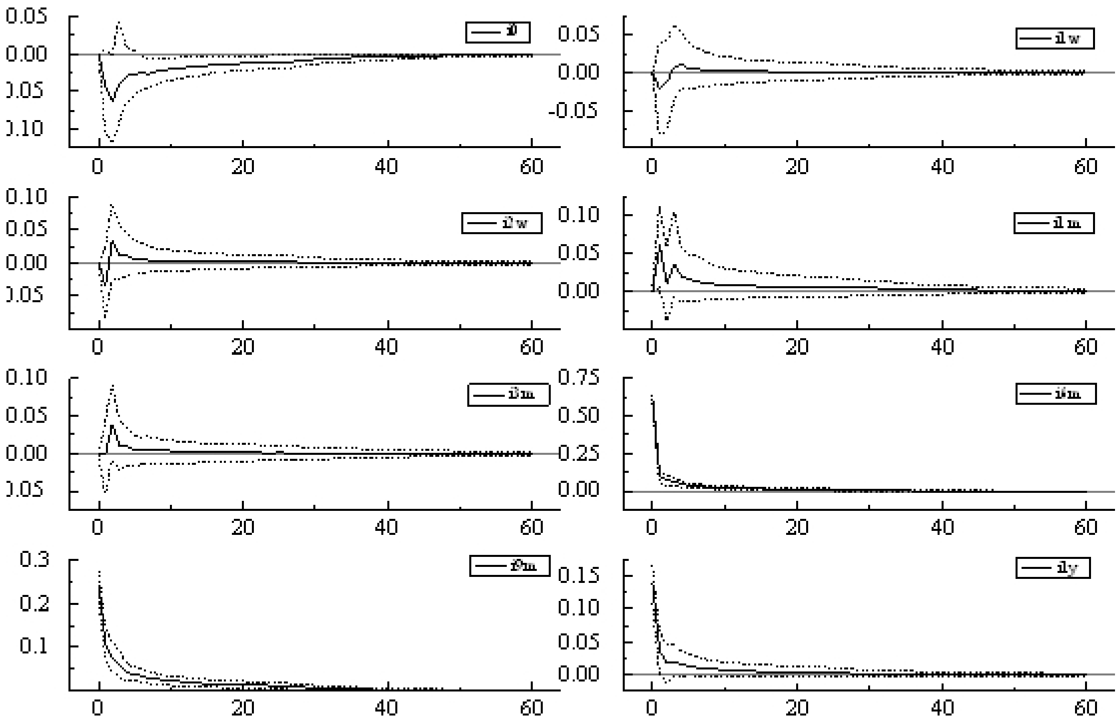

Figure 9: Impulse response functions; unitary six-month rate shock.

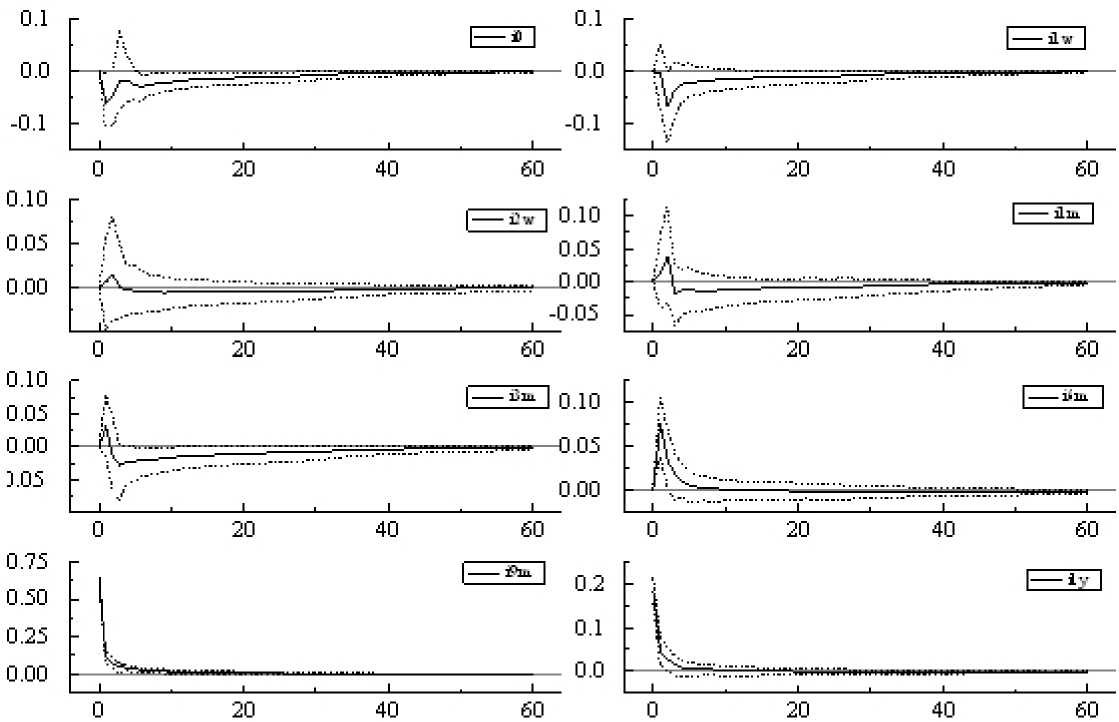

Figure 10: Impulse response functions; unitary nine-month rate shock.

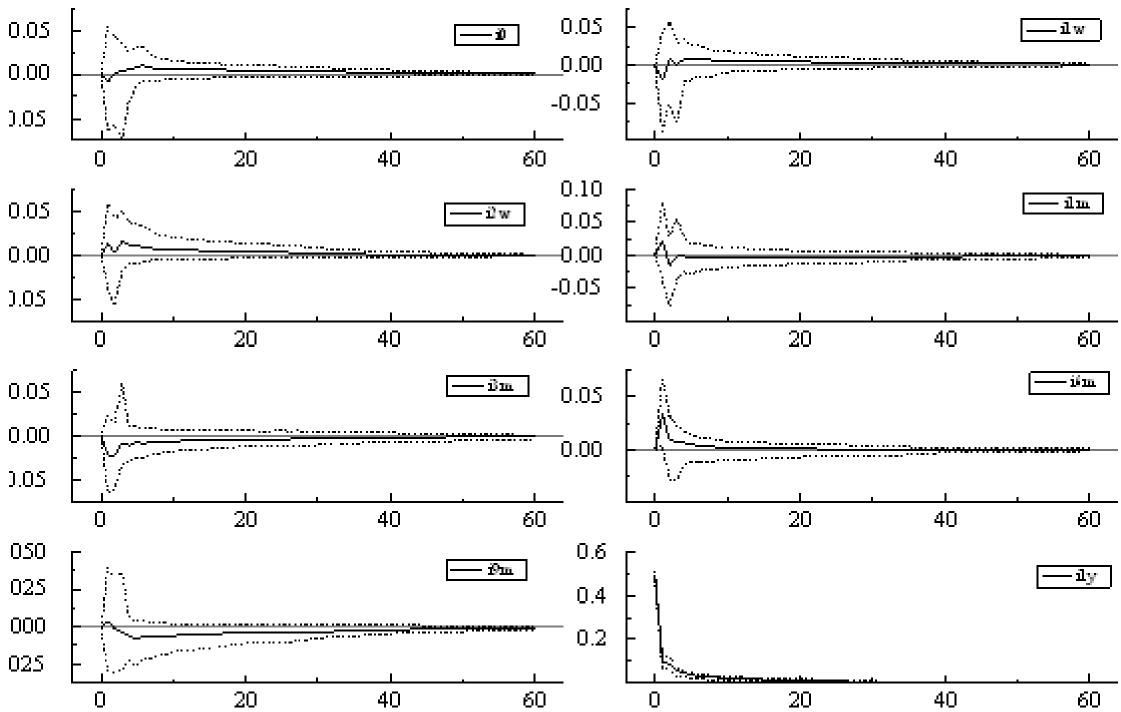

Figure 11: Impulse response functions; unitary one-year rate shock.

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