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De-anchoring of inflation expectations
under learning and heterogeneity

Task force on low inflation (LIFT)

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This paper presents research conducted within the Task Force on Low Inflation (LIFT). The task force is composed of economists from the European System of Central Banks (ESCB) - i.e. the 29 national central banks of the European Union (EU) and the European Central Bank. The objective of the expert team is to study issues raised by persistently low inflation from both empirical and theoretical modelling perspectives.

The research is carried out in three workstreams:

- 1) Drivers of Low Inflation;
- 2) Inflation Expectations;
- 3) Macroeconomic Effects of Low Inflation.

LIFT is chaired by Matteo Ciccarelli and Chiara Osbat (ECB). Workstream 1 is headed by Elena Bobeica and Marek Jarocinski (ECB) ; workstream 2 by Catherine Jardet (Banque de France) and Arnoud Stevens (National Bank of Belgium); workstream 3 by Caterina Mendicino (ECB), Sergio Santoro (Banca d'Italia) and Alessandro Notarpietro (Banca d'Italia).

The selection and refereeing process for this paper was carried out by the Chairs of the Task Force. Papers were selected based on their quality and on the relevance of the research subject to the aim of the Task Force. The authors of the selected papers were invited to revise their paper to take into consideration feedback received during the preparatory work and the referee's and Editors' comments.

The paper is released to make the research of LIFT generally available, in preliminary form, to encourage comments and suggestions prior to final publication. The views expressed in the paper are the ones of the author(s) and do not necessarily reflect those of the ECB, the ESCB, or any of the ESCB National Central Banks.

Abstract

The paper studies how a prolonged period of subdued price developments may induce a de-anchoring of inflation expectations from the central bank's objective. This is shown within a framework where agents form expectations using adaptive learning, choosing among a set of alternative forecasting models. The analysis is accompanied by empirical evidence on the properties of inflation expectations in the euro area. Our results also suggest that monetary policy may lose effectiveness if delayed too much, as expectations are allowed to drift away from target for too long.

Keywords: Learning, DSGE, Expectations de-anchoring, Inflation.

JEL classification: E31, E37, E58, D83.

Non-technical Summary

Since late 2013, euro area headline inflation has been surprising analysts and policymakers on the downside. In September of that year most institutions (including the ECB) were projecting inflation to hover around 1.5% in 2014; the realized value was instead 0.4%, with consumer prices even declining in December. Large and persistent negative forecast errors continued to be the norm throughout 2014, with market-based inflation expectations progressively adjusting downward toward actual price developments, even at long horizons. These features in the inflation dynamics have driven the ECB monetary policy since mid 2014 with the concern of inflation remaining too low for too long, viewed as a threat to the anchoring of inflation expectations.

This paper investigates how a prolonged period of subdued price developments may induce a de-anchoring of expectations from the central bank's objective. Using survey data of the inflation expectations in the euro area, we provide evidence of adaptive learning and heterogeneity in the expectations formation mechanism. We embed this evidence in a small-scale New Keynesian model assuming that the agents form their expectations by adaptive learning and choose the preferred forecasting rule within a given set of models. In our baseline experiment we assume that there exist just two competing forecasting models: one takes into account ('trust') the central bank's objective, while the other (naive) uses present and past inflation's realizations only. The share of agents adopting each model evolves according to the relative forecasting accuracy.

In such an environment, a sequence of deflationary shocks can trigger a de-anchoring of inflation expectations in two ways: by reducing the perceived inflation target and by increasing the share of agents selecting the naive model. The resulting gap between target and actual inflation can be persistent, with actual inflation recovering very slowly toward the central bank target associated with a prolonged period of negative output gap.

Our results suggest that even the adoption of bold monetary policy actions may lose effectiveness if delayed too much, as expectations are allowed to drift away from target for too long.

1 Introduction

Since late 2013, euro area headline inflation has been surprising analysts and policymakers on the downside. In September of that year most institutions (including the ECB) were projecting inflation to hover around 1.5% in 2014; the realized value was instead 0.4%, with consumer prices even declining in December. Large and persistent negative forecast errors continued to be the norm throughout 2014, with market-based inflation expectations progressively adjusting downward toward actual price developments, even at long horizons.¹

Such inflation developments have driven the ECB monetary policy since mid 2014: the targeted long-term refinancing operations and the negative interest rate on the deposit facility (in June), the ABS and covered bond purchase programmes (in September) and the public sector purchase programme (in January 2015) were all motivated by concerns of inflation remaining too low for too long, viewed as a threat to the anchoring of inflation expectations.

If the assumption of rational expectation (RE) holds and the central bank is fully credible, though, long-term inflation expectations are tied to the central bank inflation target. Such strong conclusion does not necessarily hold if agents have limited information about the working of the economy and/or display learning-type behaviour.² Models with learning agents have been considered, in the context of monetary policy, by Orphanides and Williams (2005, 2007), Milani (2007) and Gaspar et al. (2011), among others.³

The purpose of this paper is to investigate how a prolonged period of subdued price developments may induce a de-anchoring of expectations from the central bank's objective. First, using survey data we provide evidence for adaptiveness and heterogeneity of inflation expectations in the euro area, finding an increased

¹For a comprehensive account of inflation developments in the Euro area after 2013 and the ensuing policy responses see Neri and Siviero (2015).

²An alternative avenue to escape the conclusion is to appeal to sunspot models characterized by multiple self-fulfilling equilibria. For a recent attempt to explain "Japan lost decade" and the current Euro area deflationary environment along these lines, see Piazza (2015) and Buono and Piazza (2015).

³Early work on learning expectations are Bray (1982), Evans (1985) and Marcet and Sargent (1989a,b). Evans and Honkapohja (2001) is a comprehensive introduction to learning expectations.

sensitivity of beliefs to current developments in more recent periods. Then, we embed this evidence in a small-scale New Keynesian model by positing that agents have limited information and form expectations through an adaptive learning process. Every period agents choose their preferred forecasting model within a given set: some explicitly take into account the central bank's target while others use a more 'naïve' approach. In our baseline experiment we assume that there exist just two competing forecasting models: one closely resembling the RE solution; the other more 'naive', just capturing the stickiness of inflation. The share of agents adopting each model evolves according to its relative forecasting accuracy.

We design the following experiment: first, the model's deep parameters are estimated under RE using data only up to 2012; second, using a filtering technique, we recover the structural shocks that align the model dynamics with the more recent data and forecasts (up to 2017); finally, we assess the occurrence of de-anchoring of inflation expectations by re-simulating the model under alternative expectations formation mechanisms, keeping fixed the sequence of structural shocks. The main findings are the following:

1. as in Slobodyan and Wouters (2012a) when agents use a learning model whose specification is close to the RE solution, the resulting dynamics tracks reasonably well that prevailing under RE. This is no longer the case if learning is based on simple univariate processes;
2. under heterogeneous expectations a sequence of negative shocks may bring actual inflation off target in two ways: it reduces the perceived inflation target (mean inflation effect), and it increases the share of agents selecting the 'naive' PLM (model selection effect). Moreover, when agents use naive forecasting models for both inflation and output, the de-anchoring is more severe due to a self-reinforcing mechanism;
3. the gap between target and actual inflation can be very persistent, with actual inflation recovering only very slowly toward the central bank target.

Our work is related to several areas of research. The policy implications of monetary models under learning has been studied in Orphanides and Williams (2005) and Orphanides and Williams (2007). In particular, they showed that learning

leads to a bias towards more “hawkish” policies, and persistent deviations of inflation expectations from target can arise from a sequence of unfavorable shocks.⁴ The use of learning expectations has also been advocated on empirical grounds as an alternative to the assumptions of habits (in consumption) and indexation use to match the persistence of the data; see Milani (2007) and Slobodyan and Wouters (2012a,b). Models of learning under heterogeneous expectations have been typically used in the literature to obtain richer dynamics. Brock and Hommes (1997) introduces heterogeneity in adaptive expectations to describe high-order cycles and chaotic dynamics. De Grauwe (2012) follows a setup similar to ours to analyze boom and bust cycles. In our case, heterogeneous inflation expectations, other than being empirically plausible, are crucial to explain the persistent departure of inflation from the central bank target we see in the data.

The paper proceeds as follows. Section 2 provides empirical evidence based on survey data on the dispersion and evolution of inflation expectations in the euro area, consistent with the view that they are the outcome of an adaptive learning mechanism. The model is presented in Section 3. Results on the impact of a prolonged series of deflationary shocks are showed in Section 4. Some robustness checks are provided in Section 5 and Section 6 concludes.

2 Empirical evidence of adaptive learning and heterogeneity in the inflation expectations

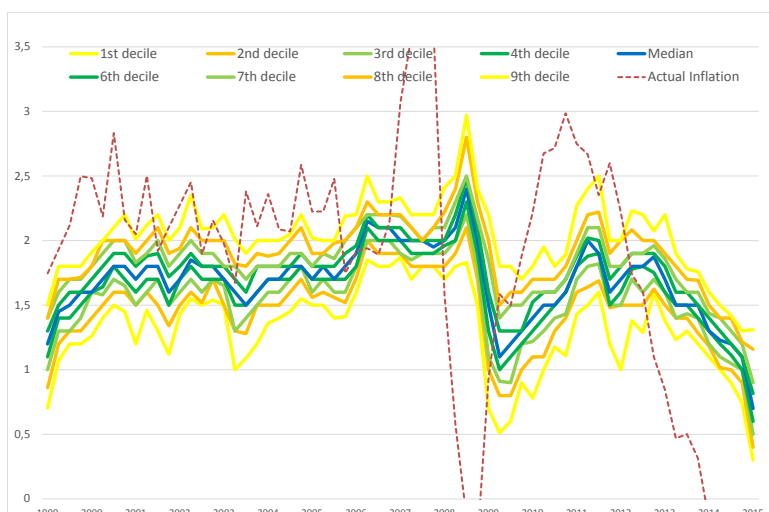
Euro area inflation expectations have progressively declined since the second half of 2013, moving away from the ECB target even at the longer horizons. This reduction mirrors the effect of a prolonged period of low inflation and weak economic activity, where inflation data have most of the time surprised on the downside. Indeed, Miccoli and Neri (20015) find that market-based inflation expectations respond (with a positive coefficient) to current inflation surprises, conditioning on cyclical conditions, and commodity prices. This result can not hold under RE hypothesis, perfect information and fully credible central bank, but it is instead

⁴Bullard and Mitra (2007) and Evans and McGough (2005) investigate the equilibrium properties of models with adaptive learning.

consistent with the assumption of adaptive learning expectations.

Looking at survey data, most empirical studies indicate that inflation expectations do not appear fully rational. For the US, using a long span of data, Thomas (1999) finds that inflation expectations - taken from various surveys - appear unbiased, but he rejects the hypothesis of efficiency (in the sense of orthogonality with respect to the available information); analogous results are obtained by Forsells and Kenny (2004) for the euro area using the European Commission survey. Mankiw et al. (2004) instead can reject both hypotheses of unbiasedness and efficiency. Empirical evidence in favour of adaptive learning in inflation expectations is provided, among else, by Branch (2004) and Pfajfar and Santoro (2010). Survey data also show that there is a lot of heterogeneity (or 'disagreement') in agents' expectations; see Mankiw et al. (2004), Branch (2004, 2007) for the US. In general, heterogeneity appears stronger among consumers than economists or professional forecasters.

Figure 1: Cross-section distribution of 1-year ahead SPF inflation forecasts

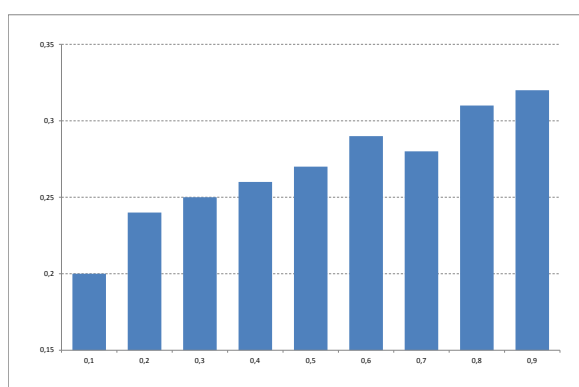


In this section we provide some novel empirical evidence on adaptiveness and heterogeneity of expectations of euro area inflation, using the ECB Survey of Professional Forecasters (SPF).⁵ Figure 1 shows the deciles of the cross-section distri-

⁵The survey is run by the ECB since 1999 at a quarterly frequency. It collects data on expect-

bution of one year ahead point forecasts of the SPF participants together with the realized value of inflation; quarterly data over the period 1999Q1-2015Q1. Heterogeneity in inflation expectations appears to be substantial and time-varying: measured in terms of the difference between the 9th and the 1st decile, it ranges from about 0.5 to 1.5%. Inflation forecasts (and forecast errors) tend to be persistent, reflecting the current developments: indeed, the correlation between the one-year ahead median forecast and the current level of inflation is 0.83.

Figure 2: Error-adjustment coefficient across agent-types



Following the approach of Pfajfar and Santoro (2010) we analyze the different percentiles of the cross-sectional distribution of the SPF inflation forecasts, interpreting them as expectations coming from the same "agent-type". In this way various assumptions regarding the adaptiveness of expectations across types of agents can be tested.⁶

Let $\pi_{t+h|t}^i$ be the h -step ahead inflation forecast made at time t by agent-type i . We start by considering the simple adaptive expectations rule,

$$\pi_{t+h|t}^i = \pi_{t|t-h}^i + \alpha (\pi_t - \pi_{t|t-h}^i)$$

tations (at various horizons) for inflation, GDP growth and unemployment in the euro area. The participants are experts affiliated with financial or non-financial institutions based within the European Union. See <http://www.ecb.europa.eu/stats/prices/indic/forecast/html/index.en.html> for the details.

⁶Pfajfar and Santoro (2010) studied the data from Miching survey of consumers while we consider the euro area Survey of Professional Forecasters

by which the agent-type i revises its expectation according to the last observed forecast error; i refers to the deciles of the cross-section distribution and h is the one-year ahead horizon. The parameter α , the error-adjustment coefficient, encodes the speed of adjustment of the current forecast to the past error. The OLS estimate of α is showed in Figure 2 for agent-types: its value ranges between 0.20 for the lowest decile to 0.32 for the highest. Consistently with the findings of Pfajfar and Santoro (2010), we find that the degree of adaptiveness is higher in the right hand side of the distribution, whereas inflation forecast are more persistent in the lower deciles.

In the simple adaptive formula written above the adjustment coefficient to past errors is assumed constant over time. In a more general setting the weights attached to the observed data of inflation are time-varying. Consistently with these ideas, we fit the following time-varying coefficients model to the deciles of the distribution of expected inflation

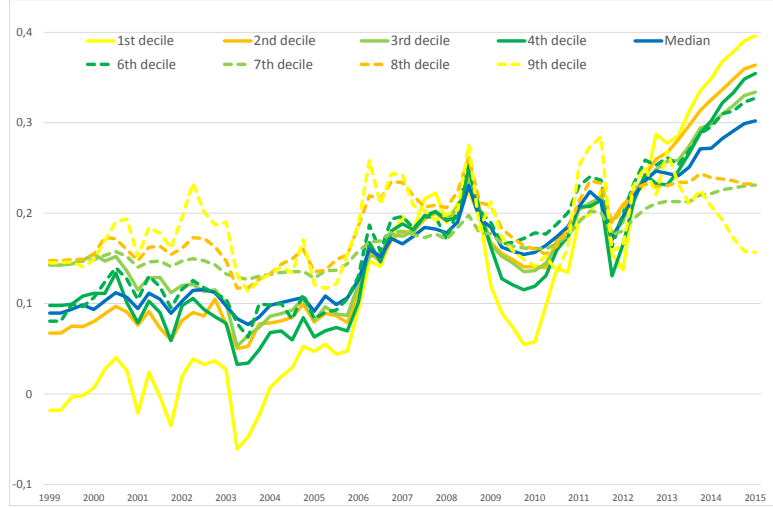
$$\pi_{t+h|t}^i = \alpha_{0,t} + \alpha_{1,t}\pi_t + \varepsilon_t,$$

where $\alpha_{0,t}$, $\alpha_{1,t}$ are assumed to evolve as Gaussian random walk processes and ε_t is a Gaussian white noise disturbance. The model is handled with state-space techniques as described in Harvey (1989). The estimates of the time-varying coefficient $\alpha_{1,t}$ attached to the current level of inflation are provided in Figure 3 across agent-types.

Three interesting facts emerge from the graph. First, for each agent-type the coefficient tends to be fairly stable during the Great Moderation period 1999-2006 whereas it fluctuates widely thereafter. Second, there is a lot of heterogeneity in the weights to observed inflation across agents; note that in the average of 1999-2015, the weight is larger for the agent-types belonging to the right hand side of the distribution, which is consistent with the estimates reported in Figure 2. Third, since 2013 there appears to be a break in the mechanism of expectations formation, placing more weight on the current level of inflation. This is more evident for the agent-types in the lower part of the distribution.

These findings corroborate the model-based analysis conducted in the remainder of the paper, where we assume adaptive learning expectations and heterogeneous beliefs.

Figure 3: Time-varying coefficient on current inflation across agent-types



3 Theoretical setup

We consider a simple version of a New Keynesian model, featuring nominal price rigidities and intrinsic inertia in inflation and output, as in, inter alia, Clarida, Gali and Gertler (1999). The linearized model is given by:

$$\pi_t = \psi E_t \pi_{t+1} + (1 - \psi) \pi_{t-1} + \kappa x_t + \varepsilon_{\pi,t} \quad (1)$$

$$x_t = \lambda E_t x_{t+1} + (1 - \lambda) x_{t-1} - \varphi (i_t - E_t \pi_{t+1} - \bar{r}) + \varepsilon_{x,t} \quad (2)$$

$$i_t = \max\{0, \rho i_{t-1} + (1 - \rho) \bar{i} + \alpha_\pi (\pi_t - \bar{\pi}) + \alpha_x x_t + \varepsilon_{i,t}\} \quad (3)$$

where π_t is the inflation rate, x_t the output gap, i_t the monetary policy interest rate, $\bar{\pi}$ the central bank inflation target, and \bar{i} (\bar{r}) the equilibrium nominal (real) interest rate; E_t indicates expectations formed at time t .

Equation (1) represents the Phillips curve, (2) is the IS curve and (3) the interest rate rule; the corresponding supply, demand and monetary policy shocks are $\varepsilon_{\pi,t}$, $\varepsilon_{x,t}$ and $\varepsilon_{i,t}$ which are assumed to be AR(1) processes, to account for the persistence in the data. When the model is simulated under learning the $\max\{\cdot\}$ operator in (3) avoids negative realizations of the policy rate, thus explicitly ac-

counting for the zero lower bound (ZLB).⁷

Under RE, economic agents know the model, the parameters and the shocks up to time t and use this knowledge to form expectations of inflation and output. Using the *minimum state variable* (MSV) representation, the solution of the model can be written as

$$s_{t+1} = Gs_t + H\varepsilon_t \quad (4)$$

$$E_t y_{t+1} = Fs_t \quad (5)$$

where s_t is a vector containing the (minimum number of) state variables and y_t collects the forward-looking variables whose future value has to be anticipated; G , H and F are matrices of coefficients which depend on the structural parameters. The vector s_t typically includes both observable as well as unobservable variables and shocks.

Once the RE assumption is dropped, the learning mechanism employed by economic agents to form expectations has to be specified (the expectations operator under learning will be denoted by \hat{E}_t). In the experiment described in the next section we assume that agents behave like econometricians and use regression equations whose specification can vary but in general has the following form:

$$\tilde{w}_t = B_{t-1}\tilde{w}_{t-1} + v_t \quad (6)$$

where \tilde{w}_{t-1} typically includes only the subset of states that are observables. In our case \tilde{w}_{t-1} contains: lagged inflation, lagged estimate of the output gap and lagged interest rate, plus a constant, i.e. $\tilde{w}_t = (1, x_t, \pi_t, i_t)'$. B_{t-1} is a matrix of coefficients updated every period: specifically, each (relevant) row b_{t-1}^j of B_{t-1} is updated using:

$$\begin{aligned} b_t^j &= b_{t-1}^j + \gamma R_{t-1}^{-1} \tilde{w}_{t-1} (y_t^j - \tilde{w}_{t-1}' b_{t-1}^j), \quad j = x, \pi, i, \quad y_t^j = x_t, \pi_t, i_t, \\ R_t &= R_{t-1} + \gamma (\tilde{w}_{t-1} \tilde{w}_{t-1}' - R_{t-1}), \end{aligned} \quad (7)$$

⁷Under ZLB the system therefore collapses to a two equations model; see details in the appendix. On the other hand, the model under RE is estimated for the period 199Q1-2012Q4 and the interest rate is positive by construction.

where γ is the so called *constant gain* parameter. Under this learning scheme the most recent observations receive relatively higher weights, while in the standard *decreasing gain* (least-square) learning, where γ is replaced by the term $(\frac{1}{t})$, each observation even from the distant past is equally weighted (see e.g. Evans and Honkapohja 2001). Equation (6) serves as the *perceived law of motion* (PLM) and implies that the expectation under learning is

$$\hat{E}_t \tilde{w}_{t+1} = B_t \tilde{w}_t. \quad (8)$$

When the economy is described by (1)-(3), it is not sensible to assume that agents know the current realization of \tilde{w}_t when forming expectations for \tilde{w}_{t+1} ; therefore we posit that agents need to iterate on the PLM (6) to form expectations for time $(t + 1)$ based on information at time $(t - 1)$:

$$\hat{E}_{t-1} \tilde{w}_{t+1} = B_{t-1}^2 \tilde{w}_{t-1}. \quad (9)$$

Replacing the expectation terms in (1)-(3) by means of (9), we obtain that the *actual law of motion* (ALM) is a VAR with drifting coefficients; see details in the Appendix.

When the forecasting equation is not the same for all agents, two additional problems arise, namely: (i) which variables enter the PLMs? (ii) how do agents switch from one forecasting rule to another?

3.1 Heterogeneous expectations

If agents can pick one out of a large number of forecasting equations, none of which is clearly superior, expectations are heterogeneous, since nothing guarantees that everyone will choose the same PLM; on top of that, agents may elect to change their forecasting equation if they perceive its accuracy to be poor; third, several PLMs can survive and coexist asymptotically.

In our baseline specification, we assume that agents can choose between two forecasting models (PLMs). The first one, which we call the *anchored* (A) model, is more sophisticated as it is closer to the MSV solution under RE. In particular,

the set of equations describing expectations formation (PLM) under the A model is formalized as follows:

$$\begin{aligned}
\widehat{E}_{t-1}^A \pi_t &= b_{1,t-1}^\pi (x_{t-1} - \bar{x}) + b_{2,t-1}^\pi (\pi_{t-1} - \bar{\pi}) + b_{3,t-1}^\pi (i_{t-1} - \bar{i}) + \bar{\pi}, \\
\widehat{E}_{t-1}^A x_t &= b_{1,t-1}^x (x_{t-1} - \bar{x}) + b_{2,t-1}^x (\pi_{t-1} - \bar{\pi}) + b_{3,t-1}^x (i_{t-1} - \bar{i}) + \bar{x}, \\
\widehat{E}_{t-1}^A i_t &= b_{1,t-1}^i (x_{t-1} - \bar{x}) + b_{2,t-1}^i (\pi_{t-1} - \bar{\pi}) + b_{3,t-1}^i (i_{t-1} - \bar{i}) + \bar{i},
\end{aligned} \tag{10}$$

where the coefficients $b_{i,t}^j$ are updated every period using *constant gain* type-learning (7) consistent with the specification in (10). This first model is richly parametrized, as it includes all the observable state variables of the MSV solution, and requires agents to know (or believe in) the unconditional means (targets) values of the variables ($\bar{\pi}$, \bar{x} and \bar{i}). Thus it requires agents to have the largest information set and, under fairly mild conditions about the learning process and the law of motion of the exogenous shocks, is guaranteed to converge to the RE equilibrium.

We then introduce a more naïve PLM, called the *de-anchored* (D) model, which departs from the first one in two directions: (i) the agents do not know (or do not believe) the targets (unconditional means), which are replaced by time-varying levels; (ii) the inflation forecasting rule does not include past values of output and interest rate: i.e. inflation expectation follows a first-order autoregressive model. The equations describing the PLM under the D model are:

$$\begin{aligned}
\widehat{E}_{t-1}^D \pi_t &= \tilde{b}_{0,t-1}^\pi + \tilde{b}_{1,t-1}^\pi \pi_{t-1}, \\
\widehat{E}_{t-1}^D x_t &= \tilde{b}_{0,t-1}^x + \tilde{b}_{1,t-1}^x x_{t-1} + \tilde{b}_{2,t-1}^x \pi_{t-1} + \tilde{b}_{3,t-1}^x i_{t-1}, \\
\widehat{E}_{t-1}^D i_t &= \tilde{b}_{0,t-1}^i + \tilde{b}_{1,t-1}^i x_{t-1} + \tilde{b}_{2,t-1}^i \pi_{t-1} + \tilde{b}_{3,t-1}^i i_{t-1}.
\end{aligned} \tag{11}$$

As above, the coefficients $\tilde{b}_{i,t}^j$ are updated using *constant gain* learning rule applied to the PLM (11). This second model is more parsimonious, hence less prone to overfitting when estimated recursively, and requires less information from agents. Simple autoregressive models of this kind have been shown to have a very good track record when it comes to forecast inflation (Cogley 2002, Faust and Write 2013) and to produce forecasts in line with survey evidence on inflation expecta-

tions (Slobodan and Wouters 2012a). Actual dynamics under it, though, are not guaranteed to converge to any given equilibrium.

Another crucial difference is that, in the case of inflation, the sophisticated PLM allows expectations to deviate from target only in the short run, while the naive PLM does not assume full central bank credibility and lets the data speak for itself.⁸ We will show that under the naive PLM we obtain a "de-anchoring" of both expected and realized inflation.

When these two competing PLMs coexist in the population, the *aggregate* expectations for output and inflation that go into the structural model (3) are:

$$\begin{aligned}\widehat{E}_{t-1}x_t &= n_{t-1}^x \widehat{E}_{t-1}^A x_t + (1 - n_{t-1}^x) \widehat{E}_{t-1}^D x_t \\ \widehat{E}_{t-1}\pi_t &= n_{t-1}^\pi \widehat{E}_{t-1}^A \pi_t + (1 - n_{t-1}^\pi) \widehat{E}_{t-1}^D \pi_t,\end{aligned}\tag{12}$$

where n_{t-1}^x and n_{t-1}^π are the shares of agents adopting the anchored model in each case. The dynamics of these shares will be modeled in subsection 3.2.

The perceived long-run inflation is equal to the target for agents using the sophisticated (anchored) model, i.e. $\widehat{E}_{t-1}^A \bar{\pi}_t = \bar{\pi}$, while for the "naive" model this is equal to $\widehat{E}_{t-1}^D \bar{\pi}_t = \frac{\tilde{b}_{0t-1}^\pi}{1 - \tilde{b}_{1t-1}^\pi}$. Therefore, the aggregate perceived target is

$$\widehat{E}_{t-1}\bar{\pi}_t = n_{t-1}^\pi \bar{\pi} + (1 - n_{t-1}^\pi) \frac{\tilde{b}_{0t-1}^\pi}{1 - \tilde{b}_{1t-1}^\pi},\tag{13}$$

i.e. it is the linear combination of target and (discounted) average inflation. Any inflation shock will therefore affect expectations in two different ways: first, by modifying the discounted average inflation (mean inflation effect); second, by inducing agents to switch from one forecasting model to another (model selection

⁸Although the structural model (3) does not contain the expectation for the interest rate we specify a forecasting rule for it since we use iterated expectations and we need a squared matrix of coefficients B_{t-1} in (9).

effect). This is formalized as follows:

$$\begin{aligned}
\frac{\partial \widehat{E}_t \bar{\pi}}{\partial \varepsilon_{\pi, t-1}} &= (1 - n_{t-1}^\pi) \underbrace{\left[\frac{\frac{\partial}{\partial \varepsilon_{\pi, t-1}} \tilde{b}_{0t-1}^\pi (1 - \tilde{b}_{1, t-1}^\pi) + \frac{\partial}{\partial \varepsilon_{\pi, t-1}} \tilde{b}_{1t-1}^\pi \tilde{b}_{0t-1}^\pi}{(1 - \tilde{b}_{1, t-1}^\pi)^2} \right]}_{\text{mean inflation effect}} \\
&+ \underbrace{\frac{\partial n_{t-1}^\pi}{\partial \varepsilon_{\pi, t-1}} \left[\bar{\pi} - \frac{\tilde{b}_{0t-1}^\pi}{1 - \tilde{b}_{1, t-1}^\pi} \right]}_{\text{model selection effect}}
\end{aligned} \tag{14}$$

Whether the two effects reinforce each other or not depends on several factors: the persistence of the shocks, the size of the gain parameter and the sensitivity of n_{t-1}^π to predictive accuracy.

As shown in equation (14), the response of expectations is a non-linear function of the coefficients of the PLM, which in turn depend on price developments. Therefore, the learning increases the persistence of inflation because it alters its law of motion and makes more difficult for the monetary policymaker to offset the effects of inflation shocks. Intuitively, a de-anchoring of inflation expectations can happen if a sequence of negative “surprises” (i.e. a persistent deviation of actual from expected inflation) induces a downward revision of the perceived central bank’s inflation objective and an increase in the share of agents selecting the de-anchored PLM: the larger and more persistent the shocks, the stronger their macroeconomic impact through the change in the coefficients of the PLMs.

When specifying the naive PLM (11), we have assumed that agents use a simple autoregressive model for inflation, while still using all available information (vector of state variables) for output and interest rate. As a robustness check, we relax such “asymmetric behavior” and assume that agents use a first-order autoregressive model for *both* inflation and output. The implied PLM is a representation of the following *very naive* (D') model:

$$\begin{aligned}
\widehat{E}_{t-1}^{D'} \pi_t &= \check{b}_{0, t-1}^\pi + \check{b}_{1, t-1}^\pi \pi_{t-1}, \\
\widehat{E}_{t-1}^{D'} x_t &= \check{b}_{0, t-1}^x + \check{b}_{1, t-1}^x x_{t-1}.
\end{aligned} \tag{15}$$

In this case the iterated expectations $\widehat{E}_{t-1}^{D'} \pi_{t+1}$ and $\widehat{E}_{t-1}^{D'} x_{t+1}$ do not depend upon

the interest rate and thus we do not need to specify a forecasting rule for i_t . Also for this model the coefficients $\check{b}_{i,t}^j$ are updated using the algorithm in (7) applied to the PLM (15).

3.2 How agents select among different forecasting models

In order to specify a model for how n_t^x and n_t^π evolve, we resort to ideas borrowed from the evolutionary game theory literature, as we feel this is the most natural choice in a learning environment like ours. Evolutionary models exhibit learning as a primary ingredient, although typically individuals are treated as naive learners, who do not take into account that their competitors behave just like them, and thus disregard higher order beliefs.⁹

Therefore, we are going to specify a selection mechanism, known as the replicator dynamics, according to which agents tend to select strategies that do better than the population average while discarding those that do worse.

We assume a large but finite number of individuals who need to select a strategy $j \in \{1, \dots, k\}$, i.e. need to choose a forecasting model j to predict variable y_t , in a symmetric two-player game with mixed-strategy simplex in $\Delta \subset R$. Let $n_{jt} > 0$ represent the share of agents adopting each strategy/model j (we also refer to n_{jt} as the "weight" of each model in the aggregate expectation) and $\mathbf{n}_t = [n_{jt}, \dots, n_{kt}] \in \Delta$ the vector of "weights".¹⁰ The payoff to any pure predictor j (representing a pure strategy) is denoted by $g(e_{jt}, \mathbf{n}_t)$, where e_{jt} is a vector with 1 in the j^{th} position and 0 elsewhere.

The discrete-time replicator dynamics is accordingly:

$$n_{jt} = \frac{g(e_{jt-1}, \mathbf{n}_{t-1})}{\sum_{j=1}^k g(e_{jt-1}, \mathbf{n}_{t-1})}, \quad (16)$$

In the experiments described in the next section we follow De Grauwe (2012)

⁹Mailath (1998) and Samuelson (2002) are short but very good surveys of evolutionary game theory; Weibull (1995) is a comprehensive and detailed reference.

¹⁰For the vector \mathbf{n}_t to be a proper population state, it must belong to the unit simplex in (the positive region of) R .

and specify $g(e_{jt-1}, \mathbf{n}_{t-1})$ to be an exponential function of the mean-square forecast error:

$$n_{jt}^x = \frac{\exp(-\chi e_{jt-1}^x)}{\sum_{j=1}^k \exp(-\chi e_{jt-1}^x)}, \quad n_{jt}^\pi = \frac{\exp(-\chi e_{jt-1}^\pi)}{\sum_{j=1}^k \exp(-\chi e_{jt-1}^\pi)}, \quad (17)$$

where e_{jt-1}^x and e_{jt-1}^π are recursive averages of forecasting performances of model j for output and inflation respectively:

$$e_{jt}^x = (1 - \varpi)e_{jt-1}^x + \varpi(\epsilon_{jt}^x)^2, \quad e_{jt}^\pi = (1 - \varpi)e_{jt-1}^\pi + \varpi(\epsilon_{jt}^\pi)^2, \quad (18)$$

where $\epsilon_{jt}^x = (x_t - \widehat{E}_{t-1}^j x_t)$ and $\epsilon_{jt}^\pi = (\pi_t - \widehat{E}_{t-1}^j \pi_t)$ are the prediction errors made in the current quarter by model j for output and inflation respectively.

The two parameters ϖ and χ jointly determine how rapidly the vector \mathbf{n}_t , the shares of agents using each forecasting model, vary over time. The first one, ϖ , controls the window of past observations considered by the agents when assessing the forecasting performance, while the second one, χ , regulates the sensitivity of agents reacting to any given differential in the forecasting performance. Thus, for $\varpi \rightarrow 1$ and $\chi \rightarrow \infty$, agents look only at the most recent quarter and are ready to switch from one model to another based on an arbitrary small difference in forecasting performance; consequently, the vector of weights will change frequently between 0 and 1. At the other extreme, when $\varpi \rightarrow 0$ and $\chi \rightarrow 0$, the agents will "change their mind" only if there is a prolonged period of very large differences in forecasting performances; in this case, the weights will tend to remain constant and equal to the initial value. In order to have reasonable dynamics we set $\varpi = 0.25$, meaning that agents roughly look at the last four quarters of data. As for χ we experimented with different values and found that $\chi = 200$ is a reasonable value in the sense that, when one forecasting model for inflation consistently and considerably outperforms the other, it will take between six and eight periods for the entire population to switch completely to the superior PLM (see Figures 6 and 8 and the discussion in section 4.2). In any case, in section 5 we report a sensitivity analysis of our results with respect to χ .

4 Macroeconomic impact of deflationary shocks under de-anchored expectations

This section provides a model-based assessment of the impact of the persistent sequence of deflationary shocks that hit the euro area starting in 2013Q4, once we drop the assumption of RE and allow for the possibility of loosely anchored expectations.

The objective of the experiment is to assess whether a sequence of deflationary shocks can trigger a de-anchoring of inflation expectations. To do this, we compare the outcome of model simulations under two alternative expectations formation mechanisms: RE, which keep agents beliefs anchored to the central bank's objective, and adaptive learning allowing for an data-driven path of expectations.

We first estimate with Bayesian techniques the New Keynesian model (1)-(3) using euro area data over the period 1999Q1-2012Q4.¹¹ The values of the model parameters are kept fixed at the posterior mean throughout the exercises. Next, we use the Kalman filter to recover the values of the structural shocks $\varepsilon_{y,t}$, $\varepsilon_{\pi,t}$ and $\varepsilon_{i,t}$ necessary to align the dynamics of the model under RE to the more recent data (2013Q1 - 2014Q4) and forecasts (2015Q1 - 2017Q4).¹² Finally, starting from 2012Q4, we use these shocks to simulate the model under different assumptions about the expectations formation. Under RE we obtain, by construction, the original series. In this way what we refer to “data” should be interpreted as the dynamics of the system under RE.¹³

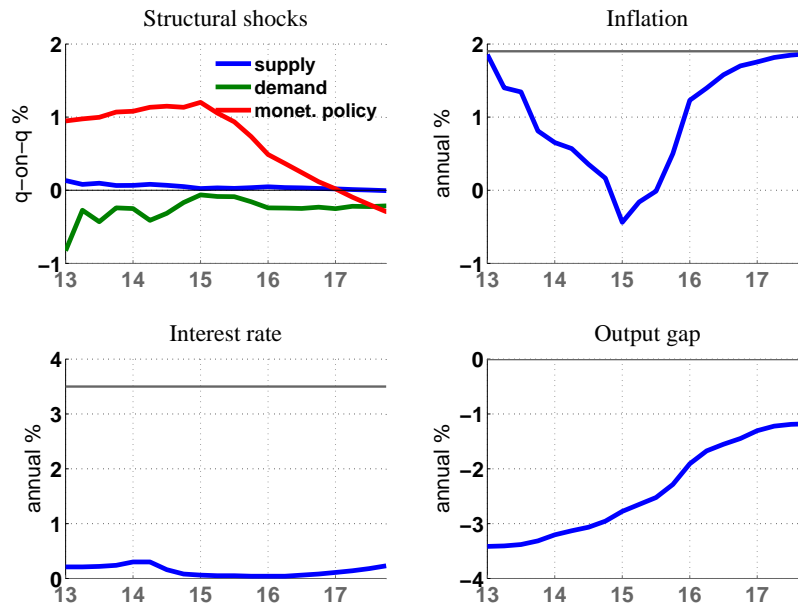
The data and the structural shocks under RE are shown in Figure 4. As expected the model attributes a marginal role to the supply (cost-push) shock $\varepsilon_{\pi,t}$, given that in the sample inflation and output tend to co-move positively: they are both low in 2013-14 and increase thereafter. The demand shock $\varepsilon_{x,t}$ is estimated

¹¹For inflation we use the year on year change in the HICP; the policy rate is proxied by the 3-month Euribor rate; the output gap estimates are taken from the ECB database.

¹² We use the Eurosystem staff projections published in the March 2015 issue of the ECB Economic Bulletin. For the output gap we employ the Eurosystem estimates but, due to confidentiality reasons, we report the OECD (publicly available) series in the figures. The output gap under learning is accordingly rescaled.

¹³Henceforth the term data is used to indicate both official statistics (up to 2014Q4) and ECB Staff projections (from 2015Q1 to 2017Q4).

Figure 4: Structural shocks under RE



to be fairly negative in the first part of the sample (2013-14), less so from 2015Q1 onward. Finally, the monetary policy shock $\varepsilon_{i,t}$ signals that the monetary stance is highly restrictive ($\varepsilon_{i,t}$ strongly positive) until 2015Q1 and then slowly normalizes. The positive values of $\varepsilon_{i,t}$ at the beginning of the sample are the mirror image of the presence of the ZLB in the data: given a very dismal performance of both inflation and output, the systematic part of the Taylor rule would have commanded a negative policy rate (an annual rate close to -4%), while in the sample the policy rate is always small but positive. After 2015Q1, when the data come from the Eurosystem official projections, the rapid decrease of the monetary policy shock reflects the rebound of inflation and of the output gap, as the projections discount the impact on the economy of the implementation of the public sector purchase programme.

4.1 Learning dynamics with homogeneous expectations

The last quarter for which we have projections is 2017Q4, which is not far enough in the future for expectations to stabilize and converge toward the long-run equilibrium. In order to have a better grasp of the persistence of the learning dynamics,

we extend the simulations by four more years, setting all structural shocks to zero for the period 2018Q1 - 2021Q4.¹⁴

The coefficients of the PLMs (i.e. of the matrix B_t) are initialized in 2013Q1 using OLS estimates over the period 1999Q1 - 2012Q4. The constant gain parameter γ in the updating rule (7) is set to $\frac{1}{40}$.

Figure 5 reports the dynamics of the model under homogeneous expectations in two different cases: in the first (red dashed line), all agents use the anchored model (10); in the second (blue dashed line), they rely upon the naïve model (11). The chart shows the dynamics of the model under heterogeneous expectations as well (black solid line), but for the moment we focus on the differences between the two cases where all agents use the same forecasting model. Initially, the two simulations look alike: as the sequence of "big" contractionary (demand and monetary policy) shocks unravels, inflation falls gradually, although not as quickly as in the "data" (gray dashed line), due to the inertia imparted by the learning mechanism. By the end of 2015, as the shocks turn less negative, a sharp difference between the two PLMs arises: under the anchored model inflation immediately rises, as the forecasting model features an unchanged "long-run mean" of 1.9% (third panel of Figure 5); on the contrary, under the naive model inflation keeps wandering at around 0.5% until the end of 2017, as the long-run mean of inflation implied by the forecasting model fails to rebound. A self-reinforcing mechanism, typical of learning environments, kicks in: lower expected inflation keeps actual price dynamics subdued, which in turn prevents expectations to recover, resulting in the decoupling of expected and target inflation.

Overall, at the end of 2017 inflation under the naive model is 1.4% lower than in either the data or the simulation under the anchored model. Interestingly, in terms of output the differences between the two models are negligible (second panel of Figure 5). This is mainly due to the fact that the two forecasting rules for output are similar, as they both use the variables entering the MSV solution as predictors, the only difference being the knowledge of - or trust in - the long-run level of output. So, even though inflation drifts downward considerably under the naive model, output dynamics is unaffected. As we will show later on, allowing

¹⁴To distinguish this part of the sample from the "data+projections" part that precedes, we use a gray-shaded background in the plots.

for a naive PLM also for the output gap leads the latter to drift away from zero.

The main message that emerges from this comparison is that when agents use a learning model resembling the MSV solution, the resulting dynamics tracks reasonably well the RE ones. A similar result is found by Slobodyan and Wouters (2012b) when studying the impact of learning in a medium-scale model a la Smets and Wouters.¹⁵

4.2 The danger of keeping inflation too low for too long

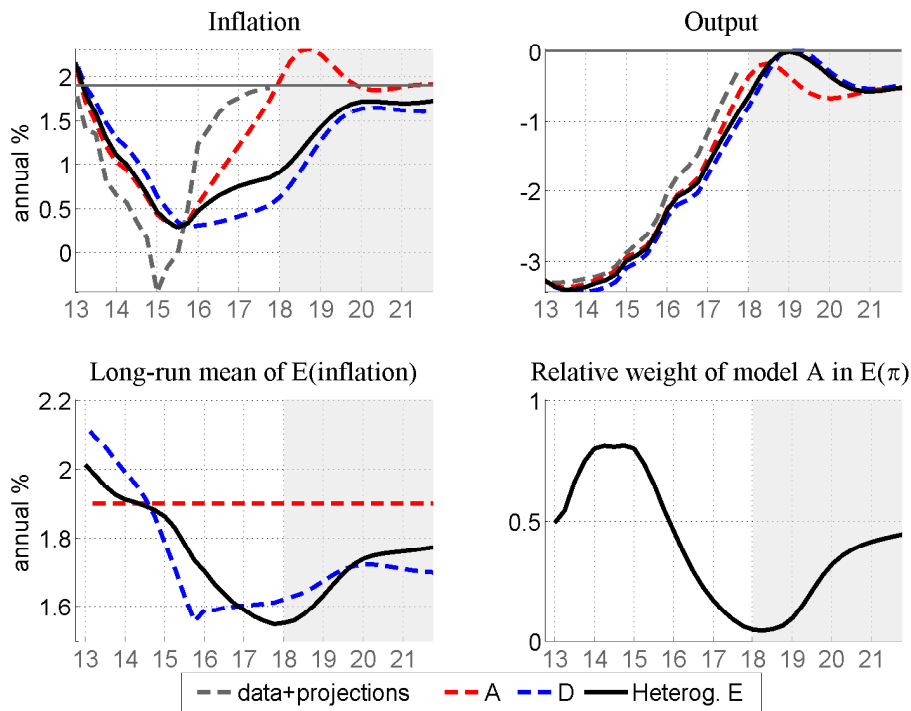
The evidence so far suggests that expectations remain well-anchored if the central bank's inflation target is credible, regardless of the expectations formation mechanism. A de-coupling between beliefs and realizations occurs only in the unlikely scenario in which all households and firms do not trust the monetary authority to act in accordance with its mandate.

A more realistic scenario is one in which agents have different beliefs, with some of them base their assessment on macroeconomic outcomes. The black solid line in Figure 5 illustrates what happens when expectations are heterogeneous and agents can switch from one forecasting model to the other according to predictive accuracy.

In the lower right panel of the figure we show the dynamics of the share of agents using the anchored model to forecast inflation. In the first period the share is initialized assuming that agents are equally distributed among the competing models; subsequently, as explained in section 3.2, the share is updated using the prediction errors and these prediction errors are shown in Figure 6. For both inflation (left panel) and output gap (right panel), we plot the forecasts made in each quarter by models (A) and (D), together with the realized values of the variable predicted. The vertical distance between the realized value and each model forecast is therefore the prediction error. The gray solid line shows the resulting aggregate expectation that enters the structural model (3).

¹⁵Slobodyan and Wouters (2012b) report that the simulated second moments under learning are not overly different from those obtained under RE if the forecasting model used by agents is close to the MSV solution; conversely, these moments diverge substantially if learning is based on simple univariate processes.

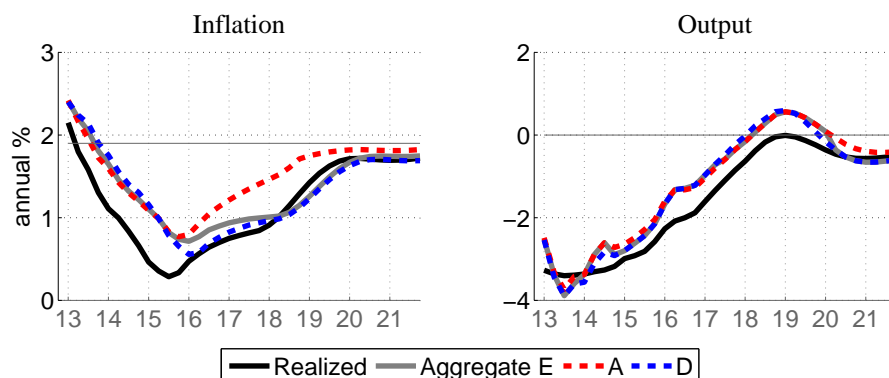
Figure 5: Homogeneous vs heterogeneous expectations - A and D models



Going back to Figure 5 we see that, initially, the dynamic under heterogeneous expectations is very similar to that of either the data or the anchored model, but in 2016 and 2017, after the sequence of negative shocks has brought actual inflation off target for a prolonged period of time, both expected and realized inflation stay below 1.0%. *The same sequence of shocks* would have brought inflation at the end of 2017 back to 1.9%, under either RE or the anchored model.

We can gain some insights looking at the evolution of model (A) share for inflation (right bottom panel of Figure 5), and at the forecasting performance of the two models (Figure 6). Initially, for inflation levels close to 1.9%, model (A) enjoys a superior performance, and its weight increasing up to 80%. As inflation keeps falling toward zero, the predictions of the naive model (D) become relative more accurate, since at low levels of inflation, the 1.9% nominal anchor embedded in model (A) introduces a substantial upward bias in predictions and deteriorates its performance. From 2015Q1 model (D) starts increasing and continuing to get larger even when the sequence of negative shocks end .

Figure 6: Models selection under heterogeneous expectations - A and D models



At the end of 2017 inflation is equal to 0.8%. Notwithstanding the adoption in 2015Q1 of the public sector purchase programme, inflation remains subdued for several years, showing that even the adoption of bold monetary policy actions may lose effectiveness if it comes too late and expectations are allowed to drift away from target for too long.

Looking at the persistence of these effects, the shaded area of the plots (where we extend the simulation until 2021Q4 setting all structural shocks to zero) show that inflation can remain low for a protracted period of time, once expectations are de-anchored. In the heterogeneous expectations scenario actual inflation recovers only slowly toward the central bank target, hovering (lingering) at around 1.7% in 2020, while the output gap closes in 2019. The persistent difference between actual and target inflation results from both a low perceived value of long-run inflation (mean inflation effect) and a comparatively large share of agents using the de-anchored PLM (model selection effect).

4.3 Using a naive model for output

There is no reason to assume that expectations are heterogeneous for inflation and homogeneous for the output gap. Figures 7 and 8 replicate the previous experiment replacing the naive model (D) with the "very naive" model (D') outlined in (15). Even though the dynamics of output gap expectations is not the focus of this paper, it can nevertheless be important, as there can be feedback loops from

one set of expectation to the other. Indeed, when we allow output expectations to follow a simple autoregressive process, the risk of a de-anchoring of inflation becomes much more severe. Specifically, from the beginning of the sample up to 2015Q3, the dynamics of inflation and output are very similar to the ones obtained previously; thereafter such substantial differences show up and inflation ends up in negative territory for much of the time during 2017-2021.

Looking at the dynamics of the weights (bottom right panel of Figure 7), the story is very similar to the previous scenario, with an initial prevalence of the anchored model, quickly reversed as inflation progressively drifts toward zero. On top of that, there is a substantial fall in output as well, with the gap still as large as -2% at the end of 2017, and not closing even after four more years. The naive model for output selected by most of the agents during the early part of the sample, which explains the difference with respect to the baseline simulation. There appears to be a self-reinforcing mechanism between the two PLMs: as the weights of the naive model for the output rises, the aggregate expected output gap decreases and this makes current inflation lower, improving the relative performance of the naive inflation model.

5 Robustness

In this section we perform robustness checks to assess how sensitive the simulation results are to model changes.

First, we assess to what extent our results depend on the degree of forward-lookingness of the Phillips curve (the parameter ψ in 1), which in our model is estimated to have a posterior mean of 0.724. The hypothesis that we want to test is whether a lower value of ψ , by decreasing the impact of expectations in the Phillips curve, reduces the role of learning and makes the assumption about the expectations formation mechanism less important.

Using the baseline case of heterogeneous expectations with PLMs (10) and (11), Figure 9 plots the dynamics of expected and realized inflation for different values of ψ , ranging from 0.2 to 0.8. The baseline calibration ($\psi = 0.724$) is shown in dashed red. There is no difference in the first part of the sample; there are, instead,

Figure 7: Homogeneous vs heterogeneous expectations - A and D' models

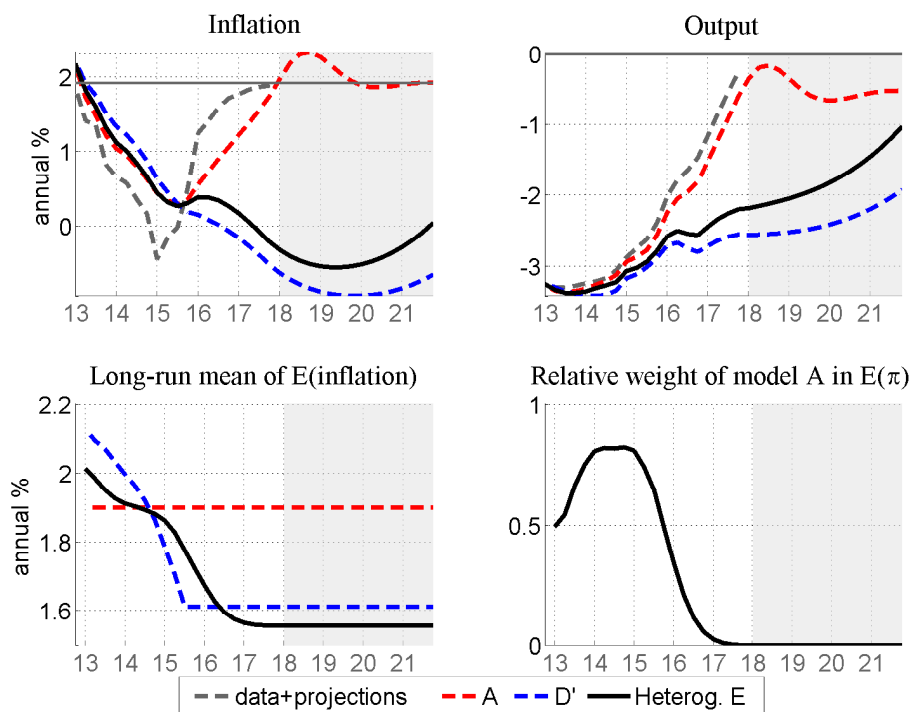


Figure 8: Models selection under heterogeneous expectations - A and D' models

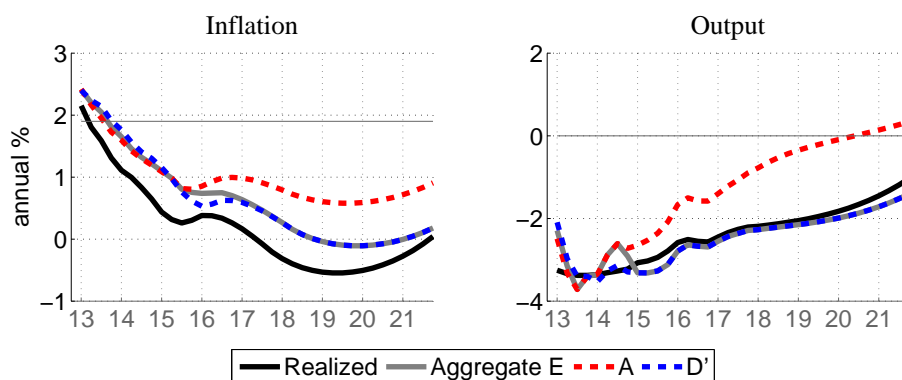
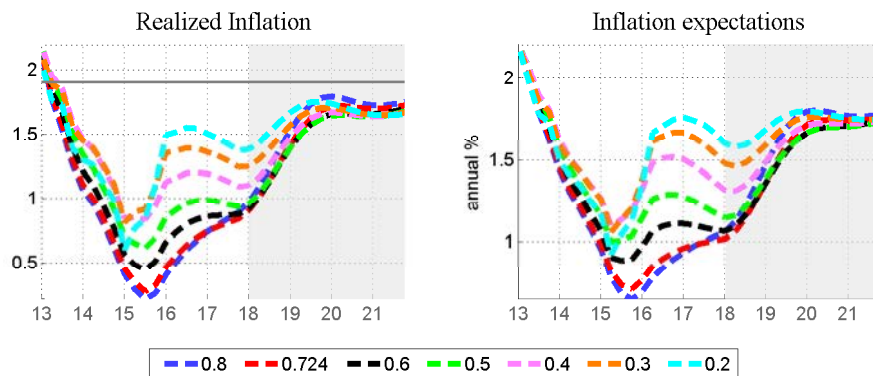


Figure 9: Heterogeneous expectations - sensitivity to the parameter ψ in the Phillips curve



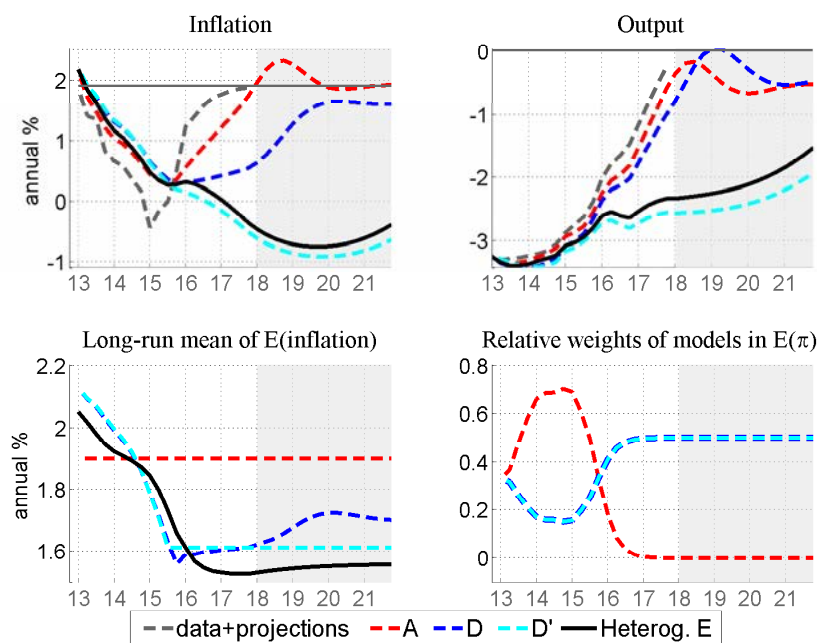
sizable discrepancies after 2015Q1, in correspondence to the rebound of inflation, when lower values of ψ lead to higher values of both expected and realized inflation. After 2016Q2 the differences however fade, as in all specifications expectations fail to converge to the central bank target. All in all, it seems that the degree of forward-lookingness in the Phillips curve is not overly relevant in driving the main results of the paper.

The second type of sensitivity analysis involves checking what happens when a larger number of forecasting models are used to form expectations. In Figure 10 we report the outcome of a scenario in which all three models previously analyzed are simultaneously present. In this case aggregate expectations are equal to:

$$\begin{aligned}\widehat{E}_{t-1}\pi_t &= \sum_j n_{jt}^\pi \widehat{E}_{t-1}^j \pi_t \\ \widehat{E}_{t-1}x_t &= \sum_j n_{jt}^x \widehat{E}_{t-1}^j x_t, \quad j = A, D, D',\end{aligned}\tag{19}$$

where n_{jt}^π and n_{jt}^x are chosen according to their relative forecasting performance as in (17)-(18). We initialize the models with equal weights. Notice that, in the case of inflation, models D and D' are indistinguishable and thus share the same evolution of the weights. The inflation dynamics seems to be very similar to that obtained in the baseline case: the anchored model dominates till 2015Q3 while the naive PLMs prevail thereafter, causing expectations and actual inflation to diverge from the central bank target. The experiment seems to confirm that the decoupling between expectations and realizations is not due to the assumption

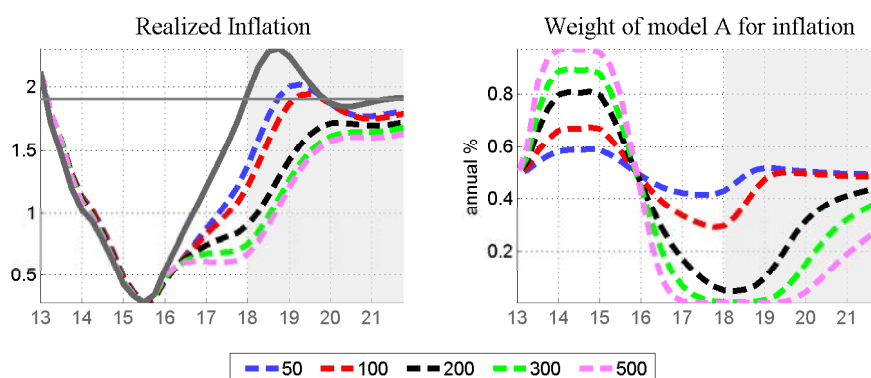
Figure 10: Homogeneous vs heterogeneous expectations - three models combined



about the number of PLMs present in the model.

As a final check, we investigate the robustness of the results with respect to the choice of the value of the parameter χ , measuring the so-called *intensity of choice*. In previous experiments it was assumed that χ was equal to 200; now values between 50 and 500 are used. Figure 11 shows realized inflation (left panel) and the dynamics of the weights on the anchored model for inflation (right panel). The chart shows that for higher values of χ the results are similar to those obtained in the baseline specification; for values lower than the baseline, the weights became very sticky, preventing agents from switching from one model to another even if their forecasting performances are considerably different. The model selection effect is therefore muted and, as a consequence, the de-anchoring is correspondingly attenuated.

Figure 11: Sensitivity to the parameter χ in the model selection. Homogeneous expectation with model A (grey solid line), heterogeneous expectations with models A and D (colored broken lines).



6 Conclusions

The paper offers a model-based assessment of the macroeconomic effects of a sequence of deflationary shocks not confronted timely with proper policy actions. Besides its theoretical interest this issue is of practical relevance for the euro area economy, as many commentators, as well as the ECB President, have related the launch in January 2015 of the asset purchase programme to the steady decline of long-term inflation expectations.

We have used a small-scale New Keynesian model in which agents do not possess the amount of information and knowledge needed to form model-consistent expectations. Agents use adaptive learning expectations to predict the future value of the variables of interest. Furthermore, the expectations formations mechanism is heterogeneous, as agents may choose among competing forecasting models. The choice is state-contingent and reflects forecast accuracy over a finite period of time. The model is specified in such a way that in principle all agents may end up choosing the same PLM, which however turns out not to be the case.¹⁶

In all the experiments the RE solution is used as a benchmark. When the assumption of full rationality is dropped, the likelihood that the economy converges

¹⁶While asymptotically one PLM is unquestionably better than the other, with a finite sample the predictive accuracy of each model is a random variable whose value depends on the sequence of shocks hitting the economy in the time frame considered for the computation of the MSEs.

to a low-growth, low-inflation equilibrium increases sizably. Under heterogeneous expectations, when some agents forecast inflation using a model that does not take the central bank's objective for granted, a sequence of negative shocks may bring inflation off target. The gap between target and actual inflation can be very persistent, and if agents use naive forecasting models for both inflation and output, the de-anchoring turns out to be much more severe due to a self-reinforcing mechanism. Even the adoption of bold monetary policy actions may lose effectiveness if delayed too much, as expectations might have drifted away from target for too long.

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7 Appendix

The model (1)-(3) has the following matrix representation

$$\Gamma_0 \tilde{w}_t = \Gamma_f E_t \tilde{w}_{t+1} + \Gamma_b \tilde{w}_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \Omega), \quad (20)$$

where $\tilde{w}_t = (1, \pi_t, x_t, i_t)'$, $\varepsilon_t = (0, \varepsilon_t^\pi, \varepsilon_t^x, \varepsilon_t^i)'$,

$$\Gamma_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\kappa & 0 \\ 0 & 0 & 1 & \sigma \\ 0 & \alpha_\pi & \alpha_x & 1 \end{bmatrix}, \quad \Gamma_f = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \psi & 0 & 0 \\ 0 & \sigma & \lambda & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (21)$$

$$\Gamma_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\lambda \bar{x} & 1 - \psi & 0 & 0 \\ \rho \bar{r} & 0 & 1 - \lambda & 0 \\ \theta & 0 & 0 & \rho \end{bmatrix}, \quad \Omega = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \sigma_\pi^2 & 0 & 0 \\ 0 & 0 & \sigma_x^2 & 0 \\ 0 & 0 & 0 & \sigma_i^2 \end{bmatrix},$$

and $\theta = (1 - \rho)\bar{i} - \alpha_\pi - \alpha_x \bar{x}$. We replace the expectations in the structural model (20) using the PLM in (9) and we obtain the following actual law of motion (ALM):

$$\tilde{w}_t = \Phi_t \tilde{w}_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \Gamma_0^{-1} \Omega (\Gamma_0^{-1})'), \quad (22)$$

where $\Phi_t = \Gamma_0^{-1} (\Gamma_f B_{t-1}^2 + \Gamma_b)$. We impose stability restrictions on the forecasting model (9), as well as on the structural model (22); in practice the roots of both B_t and Φ_t must lie inside the unit circle.

Under the ZLB the ALM (22) reduces to the following model

$$\tilde{w}_t^Z = \Phi_t^Z \tilde{w}_{t-1}^Z + \epsilon_t^Z, \quad (23)$$

where $\tilde{w}_t^Z = (1, \pi_t, x_t)'$, $\epsilon_t^Z = (0, \varepsilon_t^\pi, \varepsilon_t^x)'$, and Φ_t^Z is obtained by dropping the last column and row from the matrices in (21), as well as from B_t in the PLM (9).

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