Update and Re-estimation of NAWM II
(Version 1.4)

Günter Coenen, José-Emilio Gumiel, Carlos Montes-Galdón and Anders Warne

July 5, 2024

Abstract

This note provides a technical description of an extended version of the New Area-Wide Model (NAWM) II, which includes, as a novel element, a direct oil-price propagation channel. It first details the changes made to the specification of the previous version of the model and then reports on the estimation results for the extended model and on selected model properties.[1] In addition, the note briefly describes a newly adopted SVAR model with sign restrictions which facilitates the identification of meaningful structural shocks driving the foreign variables used in the estimation of the extended model, including the price of oil.

[1]In terms of notation and as a source for supplementary analytical derivations, the note builds on the following ECB Working Papers and mimeos:

- Christoffel, K., G. Coenen and A. Warne, 2008, "The New Area-Wide Model of the euro area: A micro-founded open-economy model for forecasting and policy analysis", ECB Working Paper No. 944.
- Coenen, G. and I. Vetlov, 2009, "Extending the NAWM with a non-zero import content of exports", ECB mimeo.
- Coenen, G., P. Karadi, S. Schmidt and A. Warne, 2018, "The New Area-Wide Model II: An extended version of the ECB's micro-founded model for forecasting and policy analysis with a financial sector", ECB Working Paper No. 2200.
- Coenen, G., J.-E. Gumiel, C. Montes-Galdón and A. Warne, 2022, "Update and re-estimation of NAWM II (Version 1.2.3)", ECB mimeo.
- Coenen, G., M. Lozej and R. Priftis, 2023, "Macroeconomic effects of carbon transition policies: An assessment based on the ECB's New Area-Wide Model with a disaggregated energy sector", ECB Working Paper No. 2819.

Table of contents

List of tables

List of figures

1 Overview

In response to the inflation surge in 2021-22 due to exceptionally large spikes in energy prices, alongside COVID 19-related factors and the Russian invasion of Ukraine, the previous version 1.2.3 of NAWM II has been extended with a direct oil price propagation channel. To this end, the aggregate consumption bundle of households has been broken down into an oil-import and a non-oil component, with a low elasticity of substitution between the two components, especially in the short term. Like the aggregate consumption bundle in the original model, the non-oil component of consumption is composed of domestic intermediate goods and imported non-oil intermediate goods, but with a higher elasticity of substitution. In addition, the model extension splits the aggregate import price Phillips curve into two separate Phillips curves for oil and non-oil import prices. This allows capturing the different speeds of pass-through of oil price changes and changes in foreign production costs to these two components of aggregate import prices.

This note provides a technical description of the extended version of NAWM II, which will be designated as version 1.4, and reports on the estimation results for the extended model and on selected model properties, including impulse response functions and historical decompositions. The note focuses on the novel elements of model version 1.4 and makes selective comparisons with version 1.2.3. To aid the identification of the separate import price Phillips-curve parameters, the oil import price deflator is used as an additional observable in the estimation of the extended model.

Another modification documented in this note concerns the adoption of a new identification approach based on sign restrictions to obtain an economically more meaningful identification of the shocks in the ancillary SVAR model capturing developments in the euro area's external environment, including oil prices.

2 Adding a direct propagation channel of oil-price changes

2.1 Demand side with oil consumption

Compared to the previous model versions, the demand side has been extended by accounting for oil consumption on the part of households.[2] The assumptions are the following:

[2] The model extension follows Coenen, Lozej and Priftis (2023), but it considers oil consumption instead of aggregate energy consumption.

- Households consume a composite final consumption good, which is composed of a consumption good excluding oil and oil imports.
- The consumption good excluding oil, and the investment and public consumption goods are composites of domestic and imported non-oil intermediate goods.
- the exported intermediate goods are composites of domestic and imported non-oil intermediate goods.[3]

Specifically, the households in the model consume a composite final good Q_t^C, which is produced by a competitive firm that bundles a consumption good excluding oil, Q_t^{CXO}, and (aggregate) oil imports for consumption purposes, IM_t^{CO},

$$Q_t^C = \left[\nu_C^{\frac{1}{\mu_C}} \left(Q_t^{CXO} \right)^{\frac{\mu_C - 1}{\mu_C}} + (1 - \nu_C)^{\frac{1}{\mu_C}} \left((1 - \Gamma_{IM^{CO},t}) IM_t^{CO} \right)^{\frac{\mu_C - 1}{\mu_C}} \right]^{\frac{\mu_C}{\mu_C - 1}}, \quad (1)$$

where μ_C is the long-run elasticity of substitution between the consumption good excluding oil and oil imports, ν_C determines their shares in the bundle, and $\Gamma_{IM^{CO},t}$ represents quadratic adjustment costs related to the change in the quantity of oil in the production of the final consumption good,

$$\Gamma_{IM^{CO},t} = \frac{\gamma_{IM^{CO}}}{2} \left(\frac{IM_t^{CO} / Q_t^C}{IM_{t-1}^{CO} / Q_{t-1}^C} - 1 \right)^2, \quad (2)$$

noting that, as it is costly to adjust the quantity of oil, the short-run elasticity of substitution between the consumption good excluding oil and oil imports can be significantly below μ_C.

The demand equations for the consumption good excluding oil and for oil imports are:

$$Q_t^{CXO} = \nu_C \left(\frac{P_{CXO,t}}{P_{C,t}} \right)^{-\mu_C} Q_t^C, \quad (3)$$

$$IM_t^{CO} = (1 - \nu_C) \left(\frac{P_{IM^O,t}}{\Gamma_{IM^{CO},t}^{\dagger} P_{C,t}} \right)^{-\mu_C} \frac{Q_t^C}{1 - \Gamma_{IM^{CO},t}}, \quad (4)$$

where $P_{C,t}$ is the price of the final consumption good, $P_{CXO,t}$ and $P_{IM^O,t}$ and are the prices of the consumption good excluding oil and oil imports, respectively, and $\Gamma_{IM^{CO},t}^{\dagger}$ denotes an analytical expression derived from the oil imports adjustment cost function (2),

$$\Gamma_{IM^{CO},t}^{\dagger} = 1 - \Gamma_{IM^{CO},t} - \Gamma_{IM^{CO},t}' IM_t^{CO}. \quad (5)$$

[3] For the extension of the original model with a non-zero import content of exports, see Coenen and Vetlov (2009).

The consumption good excluding oil is a bundle of a domestic intermediate good, H_t^C, and an imported non-oil intermediate good for consumption purposes, IM_t^{CXO}:

$$Q_t^{CXO} = \left[\nu_{CXO}^{\frac{1}{\mu_{CXO}}} \left(H_t^C \right)^{\frac{\mu_{CXO}-1}{\mu_{CXO}}} + (1-\nu_{CXO})^{\frac{1}{\mu_{CXO}}} \left((1-\Gamma_{IM^{CXO},t}) IM_t^{CXO} \right)^{\frac{\mu_{CXO}-1}{\mu_{CXO}}} \right]^{\frac{\mu_{CXO}}{\mu_{CXO}-1}}, \quad (6)$$

where μ_{CXO} is the long-run elasticity of substitution between the two goods, ν_{CXO} determines their shares in the bundle, and $\Gamma_{IM^{CXO},t}$ represents quadratic adjustment costs related to the change in the quantity of the imported non-oil intermediate good,

$$\Gamma_{IM^{CXO},t} = \frac{\gamma_{IM^{CXO}}}{2} \left(\left(\epsilon_t^{IM^{XO}} \right)^{-\frac{1}{\gamma_{IM^{CXO}}}} \frac{IM_t^{CXO} / Q_t^{CXO}}{IM_{t-1}^{CXO} / Q_{t-1}^{CXO}} - 1 \right)^2, \quad (7)$$

noting that, because of the adjustment costs, the short-run elasticity of substitution can be significantly below μ_{CXO}. $\epsilon_t^{IM^{XO}}$ represents a demand shock affecting non-oil imports uniformly across domestic demand components.

The demand equations for domestic and imported non-oil intermediate goods are:

$$H_t^C = \nu_{CXO} \left(\frac{P_{H,t}}{P_{CXO,t}} \right)^{-\mu_{CXO}} Q_t^{CXO}, \quad (8)$$

$$IM_t^{CXO} = (1-\nu_{CXO}) \left(\frac{P_{IM^{XO},t}}{\Gamma_{IM^{CXO},t}^{\dagger} P_{CXO,t}} \right)^{-\mu_{CXO}} \frac{Q_t^{CXO}}{1-\Gamma_{IM^{CXO},t}}, \quad (9)$$

where $P_{H,t}$ and $P_{IM^{XO},t}$ are the prices of the domestic and the imported non-oil intermediate goods, respectively, and $\Gamma_{IM^{CXO},t}^{\dagger}$ denotes an analytical expression derived from the non-oil imports adjustment cost function (7),

$$\Gamma_{IM^{CXO},t}^{\dagger} = 1 - \Gamma_{IM^{CXO},t} - \Gamma'_{IM^{CXO},t} IM_t^{CXO}. \quad (10)$$

The price indices corresponding to the consumption bundles Q_t^C and Q_t^{CXO} are:

$$P_{C,t} = \left[\nu_C \left(P_{CXO,t} \right)^{1-\mu_C} + (1-\nu_C) \left(\frac{P_{IM^O,t}}{\Gamma_{IM^{CO},t}^{\dagger}} \right)^{1-\mu_C} \right]^{\frac{1}{1-\mu_C}}, \quad (11)$$

$$P_{CXO,t} = \left[\nu_{CXO} \left(P_{H,t} \right)^{1-\mu_{CXO}} + (1-\nu_{CXO}) \left(\frac{P_{IM^{XO},t}}{\Gamma_{IM^{CXO},t}^{\dagger}} \right)^{1-\mu_{CXO}} \right]^{\frac{1}{1-\mu_{CXO}}}. \quad (12)$$

The production technologies for the final private investment and public consumption goods, Q_t^I and Q_t^G, and for the exported intermediate goods, X_t, as well as the corresponding intermediate-good demand equations and price indices are analogous to equations (6), (8), (9) and (12) for the consumption good excluding oil. That is, we make the simplifying assumption that these goods do not include an oil component.

Finally, we note that, for solving and simulating the model, all prices will be normalised with the price of the final consumption good, $P_{C,t}$. That is, the consumption good will serve as the numéraire.

Transformed equations

We apply the standard transformations to render the model variables stationary; see Christoffel, Coenen and Warne (2008), Appendix A, for details. Accordingly, the model equations are re-specified in terms of stationary relative prices and stationary quantities, the names of which are written using lower case letters.

The stationarity-inducing transformations of the production technology for the final consumption good (1) and the associated demand schedules (3) and (4) yield:

$$q_t^C = \left(\nu_C^{\frac{1}{\mu_C}} \left(q_t^{CXO}\right)^{1-\frac{1}{\mu_C}} + (1-\nu_C)^{\frac{1}{\mu_C}} \left((1-\Gamma_{IM^{CO},t}\, im_t^{CO}\right)^{1-\frac{1}{\mu_C}} \right)^{\frac{\mu_C}{\mu_C-1}} \tag{13}$$

and

$$q_t^{CXO} = \nu_C \left(\frac{p_{CXO,t}}{p_{C,t}}\right)^{-\mu_C} q_t^C, \tag{14}$$

$$im_t^{CO} = (1-\nu_C) \left(\frac{p_{IM^O,t}}{\Gamma^{\dagger}_{IM^{CO},t}\, p_{C,t}}\right)^{-\mu_C} \frac{q_t^C}{1-\Gamma_{IM^{CO},t}} \tag{15}$$

with $p_{C,t} = 1$ as the final consumption good serves as the numéraire, and where

$$\Gamma_{IM^{CO},t} = \frac{\gamma_{IM^{CO}}}{2} \left(\frac{im_t^{CO}/q_t^C}{im_{t-1}^{CO}/q_{t-1}^C} - 1\right)^2 \tag{16}$$

and

$$\Gamma^{\dagger}_{IM^{CO},t} = 1 - \Gamma_{IM^{CO},t} - \Gamma'_{IM^{CO},t}\, im_t^{CO}. \tag{17}$$

Similarly, the transformations of the production technology for the consumption good excluding energy (6) and the associated demand schedules (8) and (9) result in:

$$q_t^{CXO} = \left(\nu_{CXO}^{\frac{1}{\mu_{CXO}}} \left(h_t^C\right)^{1-\frac{1}{\mu_{CXO}}} + (1-\nu_{CXO})^{\frac{1}{\mu_{CXO}}} \left((1-\Gamma_{IM^{CXO},t}\, im_t^{CXO}\right)^{1-\frac{1}{\mu_{CXO}}} \right)^{\frac{\mu_{CXO}}{\mu_{CXO}-1}} \tag{18}$$

and

$$h_t^C = \nu_{CXO} \left(\frac{p_{H,t}}{p_{CXO,t}}\right)^{-\mu_{CXO}} q_t^{CXO}, \tag{19}$$

$$im_t^{CXO} = (1-\nu_{CXO}) \left(\frac{p_{IM^{CXO},t}}{\Gamma^{\dagger}_{IM^{CXO},t}\, p_{CXO,t}} \right)^{-\mu_C} \frac{q_t^{CXO}}{1-\Gamma_{IM^{CXO},t}}, \tag{20}$$

where

$$\Gamma_{IM^{CXO},t} = \frac{\gamma_{IM^{CXO}}}{2} \left(\left(\epsilon_t^{IM^{XO}}\right)^{-\frac{1}{\gamma_{IM^{CXO}}}} \frac{im_t^{CXO}/q_t^{CXO}}{im_{t-1}^{CXO}/q_{t-1}^{/CXO}} - 1 \right)^2 \tag{21}$$

and

$$\Gamma^{\dagger}_{IM^{CXO},t} = 1 - \Gamma_{IM^{CXO},t} - \Gamma'_{IM^{CXO},t}\, im_t^{CXO}. \tag{22}$$

Moreover, the transformations of the price indices for the final consumption good and the consumption good excluding oil, (11) and (12), yield:

$$p_{C,t} = \left(\nu_C \left(p_{CXO,t}\right)^{1-\mu_C} + (1-\nu_C) \left(\frac{p_{IM^O,t}}{\Gamma^{\dagger}_{IM^{CO},t}} \right)^{1-\mu_C} \right)^{\frac{1}{1-\mu_C}}, \tag{23}$$

$$p_{CXO,t} = \left(\nu_{CXO} \left(p_{H,t}\right)^{1-\mu_{CXO}} + (1-\nu_{CXO}) \left(\frac{p_{IM^{XO},t}}{\Gamma^{\dagger}_{IM^{CXO},t}} \right)^{1-\mu_{CXO}} \right)^{\frac{1}{1-\mu_{CXO}}}. \tag{24}$$

Log-linearised equations

We arrive at the following log-linearised expressions for the transformed final-good technology (13) and the transformed demand schedules (14) and (15):

$$\widehat{q}_t^C = \nu_C^{\frac{1}{\mu_C}} \left(\frac{q^{CXO}}{q^C} \right)^{1-\frac{1}{\mu_C}} \widehat{q}_t^{CXO} + (1-\nu_C)^{\frac{1}{\mu_C}} \left(\frac{im^{CO}}{q^C} \right)^{1-\frac{1}{\mu_C}} \widehat{im}_t^{CO} \tag{25}$$

and

$$\widehat{q}_t^{CXO} = -\mu_C \left(\widehat{p}_{CXO,t} - \widehat{p}_{C,t} \right) + \widehat{q}_t^C, \tag{26}$$

$$\widehat{im}_t^{CO} = -\mu_C \left(\widehat{p}_{IM^O,t} - \widehat{p}_{C,t} - \widehat{\Gamma}^{\dagger}_{IM^{CO},t} \right) + \widehat{q}_t^C, \tag{27}$$

where $\widehat{p}_{C,t} = 0$ and

$$\widehat{\Gamma}^{\dagger}_{IM^{CO},t} = -\gamma_{IM^{CO}} \left(\left(\widehat{im}_t^{CO} - \widehat{q}_t^C \right) - \left(\widehat{im}_{t-1}^{CO} - \widehat{q}_{t-1}^C \right) \right). \tag{28}$$

Similarly, we obtain the following log-linearised expressions for the transformed technology of the consumption good excluding oil (18) and the transformed demand schedules (19) and (20):

$$\widehat{q}_t^{CXO} = \nu_{CXO}^{\frac{1}{\mu_{CXO}}} \left(\frac{h^{CXO}}{q^{CXO}} \right)^{1-\frac{1}{\mu_{CXO}}} \widehat{h}_t^C + (1-\nu_{CXO})^{\frac{1}{\mu_{CXO}}} \left(\frac{im^{CXO}}{q^{CXO}} \right)^{1-\frac{1}{\mu_{CXO}}} \widehat{im}_t^{CXO} \tag{29}$$

and

$$\widehat{h}_t^C = -\mu_{CXO}\ (\widehat{p}_{H,t} - \widehat{p}_{CXO,t}) + \widehat{q}_t^{CXO}, \tag{30}$$

$$\widehat{im}_t^{CXO} = -\mu_{CXO}\ \left(\widehat{p}_{IM^{XO},t} - \widehat{p}_{CXO,t} - \widehat{\Gamma}^{\dagger}_{IM^{CXO},t}\right) + \widehat{q}_t^{CXO}, \tag{31}$$

where

$$\widehat{\Gamma}^{\dagger}_{IM^{CXO},t} = -\gamma_{IM^{CXO}}\ \left(\left(\widehat{im}_t^{CXO} - \widehat{q}_t^{CXO}\right) - \left(\widehat{im}_{t-1}^{CXO} - \widehat{q}_{t-1}^{CXO}\right)\right) + \widehat{\epsilon}_t^{IM^{XO}}. \tag{32}$$

Log-linearisation of the transformed price indices (23) and (24) results in:

$$\widehat{p}_{C,t} = \nu_C \left(\frac{p_{CXO}}{p_C}\right)^{1-\mu_C} \widehat{p}_{CXO,t} + (1-\nu_C)\left(\frac{p_{IM^O}}{p_C}\right)^{1-\mu_C} \left(\widehat{p}_{IM^O,t} - \widehat{\Gamma}^{\dagger}_{IM^C,t}\right), \tag{33}$$

$$\widehat{p}_{CXO,t} = \nu_{CXO} \left(\frac{p_H}{p_{CXO}}\right)^{1-\mu_{CXO}} \widehat{p}_{H,t} + (1-\nu_{CXO})\left(\frac{p_{IM^{XO}}}{p_{CXO}}\right)^{1-\mu_{CXO}} \left(\widehat{p}_{IM^{XO},t} - \widehat{\Gamma}^{\dagger}_{IM^{CXO},t}\right), \tag{34}$$

where $p_C = 1$.

2.2 Import demand and import prices

Oil and non-oil intermediate goods are imported from monopolistically competitive foreign producers, which set prices (in local currency) as a markup over the global price of oil in US dollar terms, $P_{O,t}$, and the foreign producer price, $P^*_{Y,t}$. That being said, we treat the specification of the demand for oil and non-oil imports and the determination of their prices in a symmetric fashion. Accordingly, we neglect the respective super and subscripts denoting the type of imports and, similarly, their use for consumption purposes.

Specifically, defining as $IM_{f^*,t}$ the differentiated import variety supplied by the foreign producer f^*, we assume that aggregate imports are a CES bundle of the import varieties:

$$IM_t = \left(\int_0^1 (IM_{f^*,t})^{\frac{1}{\varphi_t^*}}\ df^*\right)^{\varphi_t^*}, \tag{35}$$

where the possibly time-varying parameter $\varphi_t^* > 1$ is inversely related to the intratemporal elasticity of substitution between the import varieties, with $\varphi_t^*/(\varphi_t^* - 1) > 1$. The parameter φ_t^* has a natural interpretation as the markup in the markets for import varieties.

With nominal prices $P_{IM,f^*,t}$ set in monopolistically competitive markets, we obtain the following demand equations for the imported variety f^*:

$$IM_{f^*,t} = \left(\frac{P_{IM,f^*,t}}{P_{IM,t}}\right)^{-\frac{\varphi_t^*}{\varphi_t^*-1}} IM_t, \tag{36}$$

where

$$P_{IM,t} = \left(\int_0^1 \left(P_{IM,f^*,t}\right)^{\frac{1}{1-\varphi_t^*}} df^* \right)^{1-\varphi_t^*} \tag{37}$$

is the aggregate price index for the bundle of imported varieties.

Each foreign intermediate-good producer f^* sells its differentiated good in the domestic market under monopolistic competition, setting the price in local (that is, domestic) currency. There is sluggish price adjustment due to staggered price contracts à la Calvo. Accordingly, the foreign intermediate-good producer receives permission to optimally reset its price in a given period t with probability $1-\xi^*$ and has access to the following indexation scheme with parameter χ^*:

$$P_{IM,f^*,t} = \Pi_{IM,t-1}^{\chi^*} \bar{\Pi}_t^{1-\chi^*} P_{IM,f^*,t-1}, \tag{38}$$

where $\Pi_{IM,t-1} = P_{IM,t-1}/P_{IM,t-2}$ and $\bar{\Pi}_t$ denotes the (perceived) inflation target of the domestic central bank.

Each foreign producer f^* receiving permission to optimally reset its price in period t maximises the discounted sum of its expected nominal profits,

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} (\xi^*)^k \, \Lambda_{t,t+k}^* \, D_{f^*,t+k}^* / S_{t+k} \right], \tag{39}$$

subject to the price-indexation scheme (38) and the domestic import demand for its differentiated output (36), where

$$D_{f^*,t}^* = P_{IM,f^*,t} \, IM_{f^*,t} - MC_t^* \, IM_{f^*,t}, \tag{40}$$

with MC_t^* denoting the foreign producer's nominal marginal cost. S_t is the nominal exchange rate of the euro, and $\Lambda_{t,t+k}^*$ represents a suitable stochastic discount factor.

We obtain the following first-order condition characterising the foreign producer's optimal pricing decision for its output sold in the domestic market:

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} (\xi^*)^k \, \Lambda_{t,t+k}^* \left(\Pi_{IM,t,t+k}^{\dagger} \, \tilde{P}_{IM,t} - \varphi_{t+k}^* \, MC_{t+k}^* \right) \, IM_{f^*,t+k} / S_{t+k} \right] = 0, \tag{41}$$

where we have substituted the indexation scheme (38), noting that $P_{IM,f^*,t+k} = \Pi_{IM,t,t+k}^{\dagger}$ $\tilde{P}_{IM,t}$ with $\Pi_{IM,t,t+k}^{\dagger} = \prod_{s=1}^{k} \Pi_{IM,t+s-1}^{\chi^*} \bar{\Pi}_{t+s}^{1-\chi^*}$.

The associated aggregate index of price contracts for the differentiated products sold in domestic markets evolves according to

$$P_{IM,t} = \left((1-\xi^*)(\tilde{P}_{IM,t})^{\frac{1}{1-\varphi_t^*}} + \xi^* \left(\Pi_{IM,t-1}^{\chi^*} \bar{\Pi}_t^{1-\chi^*} P_{IM,t-1} \right)^{\frac{1}{1-\varphi_t^*}} \right)^{1-\varphi_t^*}. \tag{42}$$

Combining the log-linearised first-order condition for the price-setting decision of the foreign producer (41) with the log-linearised version of the corresponding aggregate import price index (42) results in the following log-linear import price Phillips curve:[4]

$$\begin{aligned}\left(\widehat{\pi}_{IM,t}-\widehat{\bar{\pi}}_{t}\right) &= \frac{\beta^{*}}{1+\beta^{*}\chi^{*}}\,\mathrm{E}_{t}\left[\widehat{\pi}_{IM,t+1}-\widehat{\bar{\pi}}_{t+1}\right]+\frac{\chi^{*}}{1+\beta^{*}\chi^{*}}\left(\widehat{\pi}_{IM,t-1}-\widehat{\bar{\pi}}_{t}\right) \\ &\quad +\frac{\beta^{*}\chi^{*}}{1+\beta^{*}\chi^{*}}\left(\mathrm{E}_{t}\left[\widehat{\bar{\pi}}_{t+1}-\widehat{\bar{\pi}}_{t}\right]\right)+\frac{\left(1-\beta^{*}\xi^{*}\right)\left(1-\xi^{*}\right)}{\xi^{*}\left(1+\beta^{*}\chi^{*}\right)}\left(\widehat{mc}_{t}^{*}+\widehat{\varphi}_{t}^{*}\right),\end{aligned} \tag{43}$$

where β^{*} is the steady-state discount factor and $\widehat{mc}_{t}^{*}$ represents the average real marginal cost of the foreign producers (expressed as logarithmic deviation from its steady-state value).

We note that, as the final domestic consumption good is used as the numéraire, the price index of the imported intermediate goods can be obtained as follows:

$$\widehat{p}_{IM,t}=\widehat{p}_{IM,t-1}+\widehat{\pi}_{IM,t}-\widehat{\pi}_{C,t}, \tag{44}$$

with $p_{IM,t}=P_{IM,t}/P_{C,t}$

Finally, recalling that the nominal marginal cost of the foreign producers of the oil and non-oil import varieties correspond respectively to the global price of oil, $P_{O,t}$, and the price of (non-oil) foreign goods, $P_{Y,t}^{*}$, we note that the respective real marginal cost are given by $\widehat{mc}_{t}^{*,O}=\widehat{s}_{t}+\widehat{p}_{Y,t}+\widehat{p}_{O,t}-\widehat{p}_{IM^{O},t}$ and $\widehat{mc}_{t}^{*,XO}=\widehat{s}_{t}+\widehat{p}_{Y,t}-\widehat{p}_{IM^{XO},t}$, where $s_{t}=S_{t}\,P_{Y,t}^{*}/P_{Y,t}$, $p_{Y,t}=P_{Y,t}/P_{C,t}$ and $p_{O,t}=P_{O,t}/P_{Y,t}^{*}$, with $P_{Y,t}$ corresponding to the domestic output deflator which is pinned down by the model's nominal aggregate resource constraint.

2.3 Aggregate resource constraint

The nominal version of the aggregate resource constraint of the extended model is given by:

$$\begin{aligned}P_{Y,t}\,Y_{t} &= P_{C,t}\,C_{t}+P_{I,t}\left(I_{t}+\Gamma_{u}(u_{t})\,K_{t}\right)+\;P_{G,t}\,G_{t}+\;P_{X,t}\,X_{t} \\ &\quad -P_{IM^{CO},t}\left(IM_{t}^{CO}\,\frac{1-\Gamma_{IM^{CO},t}}{\Gamma_{IM^{CO},t}^{\dagger}}\right) \\ &\quad -P_{IM^{XO},t}\left(IM_{t}^{CXO}\,\frac{1-\Gamma_{IM^{CXO},t}}{\Gamma_{IM^{CXO},t}^{\dagger}}+IM_{t}^{I}\,\frac{1-\Gamma_{IM^{I},t}}{\Gamma_{IM^{I},t}^{\dagger}}+IM_{t}^{X}\right).\end{aligned} \tag{45}$$

[4]The derivations, which are tedious but straightforward, are available from unpublished technical appendices accompanying the documentation of the original version of the NAWM, see Christoffel, Coenen and Warne (2008).

Applying the standard transformations and subsequent log-linearisation yields:

$$\begin{aligned}
\widehat{p}_{Y,t} + \widehat{y}_t \quad = \quad & \frac{p_C\, c}{p_Y\, y} \left(\widehat{p}_{C,t} + \widehat{c}_t\right) + \frac{p_I\, i}{p_Y\, y} \left(\widehat{p}_{I,t} + \widehat{i}_t\right) + \frac{p_I\, k\, g_z^{-1}}{p_Y\, y}\, \gamma_{u,1}\, \widehat{u}_t + \frac{p_G\, g}{p_Y\, y} \left(\widehat{p}_{G,t} + \widehat{g}_t\right) \qquad (46)\\
& + \frac{p_X\, x}{p_Y\, y} \left(\widehat{p}_{X,t} + \widehat{x}_t\right) - \frac{p_{IM^O}\, im^{CO}}{p_Y\, y} \left(\widehat{p}_{IM^O,t} + \widehat{im}_t^{CO} - \widehat{\Gamma}^{\dagger}_{IM^{CO},t}\right)\\
& - \Bigg(\frac{p_{IM^{XO}}\, im^{CXO}}{p_Y\, y} \left(\widehat{p}_{IM^{XO},t} + \widehat{im}_t^{CXO} - \widehat{\Gamma}^{\dagger}_{IM^{CXO},t}\right)\\
& + \frac{p_{IM^{XO}}\, im^{I}}{p_Y\, y} \left(\widehat{p}_{IM^{XO},t} + \widehat{im}_t^{I} - \widehat{\Gamma}^{\dagger}_{IM^{I},t}\right) + \frac{p_{IM^{XO}}\, im^{X}}{p_Y\, y} \left(\widehat{p}_{IM^{XO},t} + \widehat{im}_t^{X}\right)\Bigg),
\end{aligned}$$

recalling that $p_C = 1$ and $\widehat{p}_{C,t} = 0$.

2.4 Aggregate import volumes and prices

The nominal value of imports is given by:

$$P_{IM,t}\, IM_t = P_{IM^O,t}\, IM_t^O + P_{IM^{XO},t}\, IM_t^{XO}, \qquad (47)$$

where $IM_t^O = IM_t^{CO}$ and $IM_t^{XO} = IM_t^{CXO} + IM_t^I + IM_t^X$.

Applying the standard transformations and subsequent log-linearisation yields:

$$\widehat{p}_{IM,t} + \widehat{im}_t = \frac{p_{im^O}\, im^O}{p_{im}\, im} \left(\widehat{p}_{IM^O,t} + \widehat{im}_t^O\right) + \frac{p_{im^{XO}}\, im^{XO}}{p_{im}\, im} \left(\widehat{p}_{IM^{XO},t} + \widehat{im}_t^{XO}\right), \qquad (48)$$

where $\widehat{im}_t^O = \widehat{im}_t^{CO}$ and

$$\widehat{im}_t^{XO} = \frac{im^{CXO}}{im^{XO}}\, \widehat{im}_t^{CXO} + \frac{im^I}{im^{XO}}\, \widehat{im}_t^I + \frac{im^X}{im^{XO}}\, \widehat{im}_t^X. \qquad (49)$$

To split the nominal value of imports into volumes and prices, we consider two alternative cases. First, we assume that aggregate imports are obtained by weighing oil and non-oil imports at steady-state relative prices,

$$IM_t = \frac{P_{IM^O}}{P_{IM}}\, IM_t^O + \frac{P_{IM^{XO}}}{P_{IM}}\, IM_t^{XO}, \qquad (50)$$

and, alternatively, we assume that oil and non-oil import prices can be aggregated using a Cobb-Douglas weighing scheme,

$$P_{IM,t} = P_{IM^O,t}^{o}\, P_{IM^{XO},t}^{(1-o)}. \qquad (51)$$

The respective log-linear equations are given by:

$$\widehat{im}_t = \frac{p_{IM^O}\, im^O}{p_{IM}\, im}\, \widehat{im}_t^{\,O} + \frac{p_{IM^{XO}}\, im^{XO}}{p_{IM}\, im}\, \widehat{im}_t^{\,XO} \tag{52}$$

and

$$\widehat{p}_{IM,t} = o\, \widehat{p}_{IM^O,t} + (1-o)\, \widehat{p}_{IM^{XO},t}. \tag{53}$$

2.5 Steady-state computation

The extension of the model with a direct oil-price propagation channel results in a modification of the consolidated goods-market equilibrium condition which is used in the computation of the model's steady state.

As a first step in deriving the modified goods-market equilibrium condition, we note that the following real aggregate resource constraint holds in steady state:

$$g_z^{-\alpha} k^{\alpha} N^{1-\alpha} - \psi = h^C + h^I + h^G + h^X. \tag{54}$$

The demand schedules for the bundles of domestic intermediate goods used in the production of the final consumption and the final investment good can be stated as follows:

$$\begin{aligned} h^C &= \nu_{CXO} \left(\frac{p_H}{p_{CXO}}\right)^{-\mu_{CXO}} q^{CXO} \\ &= \nu_{CXO} \left(\frac{p_H}{p_{CXO}}\right)^{-\mu_{CXO}} \nu_C \left(\frac{p_{CXO}}{p_C}\right)^{-\mu_C} c \end{aligned} \tag{55}$$

and

$$h^I = \nu_I \left(\frac{p_H}{p_I}\right)^{-\mu_I} i, \tag{56}$$

where we have used the steady-state identities $c = q^C$ and $i = q^I$.

The demand for the bundle of domestic intermediate goods used in the production of the final government consumption good is exogenous, $h^G = q^G = g$, and the schedule for the domestic intermediate-good bundle used for producing the composite intermediate goods sold abroad is given by:

$$h^X = \nu_X \left(\frac{p_H}{p_X}\frac{\varphi^X}{\varphi^H}\right)^{-\mu_X} x, \tag{57}$$

where the parameters φ^H and φ^X denote the intermediate-goods firms' markups over the marginal cost of producing goods for domestic and foreign markets, respectively.

Using the demand schedules (55) and (57), we can re-write the aggregate resource constraint (54) as

$$h^I = g_z^{-\alpha} \left(\frac{k}{N}\right)^{\alpha} N - \psi - \nu_{CXO} \left(\frac{p_H}{p_{CXO}}\right)^{-\mu_{CXO}} \nu_C \left(\frac{p_{CXO}}{p_C}\right)^{-\mu_C} c - g - \nu_X \left(\frac{p_H}{p_X}\frac{\varphi^X}{\varphi^H}\right)^{-\mu_X} x. \tag{58}$$

As the second step in deriving the modified goods-market equilibrium condition, we employ the equation specifying the final investment-good technology,

$$i^{1-\frac{1}{\mu_I}} = \nu_I^{\frac{1}{\mu_I}} \left(h^I\right)^{1-\frac{1}{\mu_I}} + (1-\nu_I)^{\frac{1}{\mu_I}} \left(im^I\right)^{1-\frac{1}{\mu_I}}, \tag{59}$$

and substitute the re-written aggregate resource constraint (58) and the demand schedule for imported intermediate goods,

$$im^I = (1-\nu_I) \left(\frac{p_{IM^{XO}}}{p_I}\right)^{-\mu_I} i, \tag{60}$$

which, using the steady-state identity $i = (g_z - 1 + \delta)/g_z\, k$, yields:

$$\begin{aligned}\left(\frac{g_z - 1 + \delta}{g_z} k\right)^{1-\frac{1}{\mu_I}} &= \\ \nu_I^{\frac{1}{\nu_I}} \Bigg(g_z^{-\alpha} \left(\frac{k}{N}\right)^{\alpha} N - \psi - \nu_{CXO} &\left(\frac{p_H}{p_{CXO}}\right)^{-\mu_{CXO}} \nu_C \left(\frac{p_{CXO}}{p_C}\right)^{-\mu_C} c - g \\ - \nu_X \left(\frac{p_H}{p_X}\frac{\varphi^X}{\varphi^H}\right)^{-\mu_X} x\Bigg)^{1-\frac{1}{\mu_I}} &+ (1-\nu_I) \left(\frac{p_{IM^{XO}}}{p_I}\right)^{1-\mu_I} \left(\frac{g_z - 1 + \delta}{g_z} k\right)^{1-\frac{1}{\mu_I}},\end{aligned} \tag{61}$$

or, equivalently,

$$\begin{aligned}\left(\frac{g_z - 1 + \delta}{g_z} k\right)^{1-\frac{1}{\mu_I}} \left(1 - (1-\nu_I)\left(\frac{p_{IM^{XO}}}{p_I}\right)^{1-\mu_I}\right) &= \\ \nu_I^{\frac{1}{\mu_I}} \Bigg(g_z^{-\alpha} \left(\frac{k}{N}\right)^{\alpha} N - \psi - \nu_{CXO} &\left(\frac{p_H}{p_{CXO}}\right)^{-\mu_{CXO}} \nu_C \left(\frac{p_{CXO}}{p_C}\right)^{-\mu_C} c - g \\ &- \nu_X \left(\frac{p_H}{p_X}\frac{\varphi^X}{\varphi^H}\right)^{-\mu_X} x\Bigg)^{1-\frac{1}{\mu_I}}.\end{aligned} \tag{62}$$

In addition, the model extension results in a set of modified steady-state relative prices. Specifically, using the price of the final consumption good as the numéraire and taking import prices as given, the relative prices of the non-oil consumption good and the domestic intermediate goods sold at home are implicitly given by:

$$1 = \nu_C \ (p_{CXO})^{1-\mu_C} + (1-\nu_C) \ (p_{IM^O})^{1-\mu_C}\,, \tag{63}$$

$$(p_{CXO})^{1-\mu_{CXO}} = \nu_{CXO} \ (p_H)^{1-\mu_{CXO}} + (1-\nu_{CXO}) \ (p_{IM^{XO}})^{1-\mu_{CXO}}\,, \tag{64}$$

and, similarly, the price of the final investment good is given by:

$$(p_I)^{1-\mu_I} = \nu_I \ (p_H)^{1-\mu_I} + (1-\nu_I) \ (p_{IM^{XO}})^{1-\mu_I}\,. \tag{65}$$

As the price of the domestic intermediate goods sold abroad is determined as a markup over marginal cost, the relative price of the domestic intermediate goods sold abroad is given by:

$$p_X = \varphi^X \left(\nu_X \left(\frac{p_H}{\varphi^H} \right)^{1-\mu_X} + (1-\nu_X)\,(p_{IM^{XO}})^{1-\mu_X} \right)^{\frac{1}{1-\mu_X}}. \tag{66}$$

Finally, the relative price of aggregate production, or output, is determined by the normalised nominal aggregate resource constraint:

$$p_Y\, y = p_H\, h + p_X\, x - p_{IM^{XO}}\, im^X, \tag{67}$$

where $y = g_z^{-\alpha} k^\alpha N^{1-\alpha}$ and

$$im^X = \nu_X \left(\frac{p_{IM^{XO}}}{p_X \varphi^X} \right)^{-\mu_x} x, \tag{68}$$

or after some transformations:

$$\begin{aligned} p_Y \quad = \quad & \Bigg\{ p_H \left(\nu_{CXO} \left(\frac{p_H}{p_{CXO}} \right)^{-\mu_{CXO}} \nu_C \,(p_{CXO})^{-\mu_C}\, c + \nu_I \left(\frac{p_H}{p_I} \right)^{-\mu_I} \left(\frac{g_z - 1 + \delta}{g_z} k \right) + g \right) \\ & + \left(p_X - p_{IM^{XO}}(1-\nu_X) \left(\frac{p_{IM^{XO}}}{p_X} \varphi^X \right)^{-\mu_X} \right) x \Bigg\} \frac{1}{g_z^{-\alpha} k^\alpha N^{1-\alpha} - \psi}. \end{aligned} \tag{69}$$

where we have made use of the identities $h = h^C + h^I + h^G$ and $h^G = q^G = g$, equations (55), (56) and (57), as well as equation (68).

3 Model estimation

3.1 Data

In the estimation of the extended version of the model we use the database from 1980 to 2019 that was already used in the estimation of model version 1.2.3.[5] Yet to aid the identifi-

[5] The inclusion of the pandemic episode in the estimation sample is likely to require additional changes to the specification of the model to capture the unprecedented nature and scale of the associated shocks

cation of the separate import price Phillips-curve parameters in the extended model, a new variable has been added, namely the extra euro area oil import deflator. The time series for this variable is not readily available in the national accounts. The methodology followed to obtain it makes use of the Eurostat monthly trade statistics for imported petroleum, petroleum products and related materials (SITC 33) which are available for volumes and values. These statistics start only in 1991, and for the period 1989-1990 we use available euro area aggregates with a narrower coverage. For the earlier period 1980-1988 the oil import deflator is extended back assuming that it follows the pattern of oil prices in euro, a reasonable assumption given the very high correlation between both variables over the available sample. In general, the use of the oil import deflator data is subject to some caveats because of methodological differences between the trade and the national accounts statistics (e.g. different accrual principles and seasonal adjustment methods) that make them not fully consistent.

Other changes in the database relate to the transformations of competitors' export prices and oil prices. As before, these variables are expressed in terms of the currency basket underlying the construction of the nominal effective exchange rate of the euro and deflated by foreign prices, but now they are demeaned, rather than using the previous approach of removing an unrestricted linear trend. In addition, for estimating the VAR model of the foreign variables (see Section 3.2) the HP-filtered series of foreign demand is now used, while the growth rate of the foreign demand series is still employed as the observable for estimating the extended structural model.

Figure 1 depicts the time series of the new and modified data for the observed variables used in the estimation of the extended model, taking into account the transformations described above.[6]

3.2 SVAR model

In the previous versions of NAWM II, the foreign variables are modelled using a Structural Vector Autoregressive (SVAR) model with the following variables: foreign prices, foreign

and is left for a future update and re-estimation of the model which will be facilitated by the increase in available post-pandemic data points.

[6] For general details on the applied data transformations, see Christoffel et al. (2008), Section 3.2.1, and Coenen et al. (2018), Section 3.1.1.

demand, foreign short-term nominal interest rate, foreign long-term nominal interest rate, oil prices and competitors' export prices. Challenges of this approach are the identification of meaningful structural shocks in the SVAR model and the understanding of their transmission to the domestic (euro area) variables in the structural model.

Previously, identification was achieved by means of a Cholesky decomposition of the estimated variance-covariance matrix of the SVAR model. Specifically, defining y_t^* as the vector that contains the foreign variables and considering the reduced-form representation of the VAR model,

$$y_t^* = B\, y_{t-1}^* + \varepsilon_t \tag{70}$$

with $\varepsilon_t \sim \mathcal{N}(0, \Sigma)$, identification is achieved by decomposing $\Sigma = LL'$ such that L is a lower triangular matrix with real and positive diagonal entries. A Cholesky decomposition of the covariance matrix is a widely used method for identifying structural shocks in SVAR models due to its simplicity, computational efficiency, and ability to produce orthogonal shocks. However, the method imposes a recursive causal structure that may not always be theoretically justified, leading to limited flexibility and difficulties to understand the economic meaning of the resulting structural shocks. Moreover, the recursive causal structure is sensitive to the ordering of the variables in the SVAR model.

In the updated and re-estimated version of NAWM II, a new identification approach using sign restrictions is introduced to provide a more meaningful economic identification of the SVAR shocks. Specifically, the SVAR model is re-estimated with two lags under the assumption that four structural shocks drive the foreign data: foreign supply, foreign demand, foreign monetary policy, and oil price shocks. Table 1 summarises the sign restrictions introduced in the model. We introduce the restrictions only for the contemporaneous impacts of the different shocks on the endogenous variables of the model. First, a foreign supply shock is assumed to increase foreign prices but to decrease foreign demand. This shock is driven by prices not related to oil, leading to a decrease in the real price of oil and an increase in competitors' export prices. In contrast, a foreign demand shock is assumed to increase both foreign prices and nominal interest rates in response to higher demand along with stronger foreign economic activity. A foreign monetary policy shock increases nominal interest rates but causes a contraction in foreign demand and prices. It is assumed that monetary policy has a stronger impact on foreign prices than on oil prices, leading to an increase in the real price of oil. Finally, an oil price shock results in an increase in the real price of oil, which also raises foreign prices due to second-round effects, but decreases foreign demand as it acts as a cost-push shock reducing foreign economic activity.

The SVAR model is estimated using the algorithm proposed by Korobilis (2022).[7] The panels at the bottom of Figure 2 show the estimated SVAR shocks. These shocks capture well, among other developments, the large drop in demand during the global financial crisis, a long sequence of negative foreign monetary policy shocks thereafter, which captures the period of low interest rates globally, and the period of low energy prices from 2015 to 2019, which contributed to the episode of persistently low inflation in the euro area.

3.3 Estimation results

Tables 2 and 3 list the prior distributions and the posterior mode values of the model's structural parameters and its shock processes and provide a comparison of the posterior mode values for the new model version 1.4 with those for the previous model version 1.2.3. In the estimation of the updated model, the direct oil price propagation channel splits the aggregate import price Phillips curve into two separate curves for oil and non-oil import prices. The parameters of the two new Phillips curves have been assigned different prior means, reflecting the belief that the speeds of pass-through of oil price changes and changes in foreign production costs to these two components of aggregate import prices differ. This is also reflected in having two underlying shock processes for the mark-up shocks to oil and non-oil import prices, denoted by $\varphi^*_{O,t}$ and $\varphi^*_{XO,t}$, with the autoregressive and standard deviation parameters assumed to have the same prior distributions.

In addition to splitting the import price Phillips curve, the aggregate consumption bundle is separated into oil imports for consumption and a consumption good excluding oil, with distinct parameters for the long-run intratemporal substitution elasticity in the log-linearised equations (25)–(27) and (29)–(31), respectively. This separation is also reflected in distinct parameters for the adjustment costs related to the oil and non-oil import content

[7] For details, see Korobilis, D., 2022, "A new algorithm for structural restrictions in Bayesian vector autoregressions", *European Economic Review*, 148, 104241. In the algorithm, the shocks in the model are treated as unobserved factors and the estimation procedure is run as for a state-space model. The structural shocks are also complemented by idiosyncratic shocks to each variable, which do not affect the other variables on impact and can be interpreted as measurement errors. In the estimation of the model, we use the same priors as Korobilis (2022), with a tight prior imposed on the variance of the idiosyncratic shocks so that they explain only a small share of the variation in the data.

of consumption, as is apparent from the log-linearised equations (28) and (32).[8]

Finally, similar to the approach for estimating the domestic price Phillips curve in previous model versions, a system prior has now been imposed on the (reduced-form) slope parameter of the wage Phillips curve, with a somewhat lower mean and standard deviation than for the system prior on the slope of the domestic price Phillips curve.

As regards the parameters of the two separate import price Phillips curves, the posterior mode of the non-oil import price Calvo parameter is somewhat larger (0.69) than the Calvo parameter of the aggregate import price Phillips curve of the previous model version (0.58), while the oil import price Calvo parameter is much smaller (0.06). This is in line with the prior belief of different speeds of import price pass-through. For the shock processes of the two import price mark-up shocks the posterior mode values of the autoregressive parameters are of comparable size, whereas the values for the standard deviations differ substantially, with the oil-related import price mark-up shock yielding a much larger value.

Concerning the parameters governing the separation of the aggregate consumption bundle, the substitution elasticity of oil is estimated to be low in the long run (0.40) and lowered further in the short run because of sizeable adjustment costs (with an elasticity of 3.81). The long-run substitution elasticity of non-oil imports in the consumption bundle excluding oil is markedly larger (2.12) yet somewhat smaller than the substitution elasticity of aggregate imports in the previous model (2.99), with the relative size of the respective adjustment cost parameters showing a similar pattern.

Otherwise, the posterior mode estimates of the updated and re-estimated model are broadly unchanged with respect to the previous model version. Some notable differences concern the estimated degree of habit formation in consumption, which is lower in the re-estimated model, and the estimated investment adjustment cost parameter, which is also lower. Furthermore, the estimated inflation response coefficient in the interest-rate rule is somewhat lower than in the previous model version. And finally, given its central role for the implied inflation-output trade-off and hence the stabilisation performance of monetary policy, it is also noteworthy that the estimated slope parameter of the price Phillips curve is somewhat larger in the re-estimated model.[9]

[8] Note that the oil import content of consumption (s_{IM^C}/s_C) is implicitly calibrated to a value equal to 4.35 percent, which is higher than the oil share in consumption according to input-output tables and intended to halfway close the gap to the overall energy share in households' consumption expenditure.

[9] By contrast, the slope parameter of the wage Phillips curve is somewhat lower (0.29) than in the

For the updated and re-estimated model, the smoothed estimates of the structural shocks are displayed in Figure 2. The time series of the estimated shocks reveal broad similarities with the shocks of the previous model version (see Coenen et al., 2022, Figure 2). Notable exceptions relate to the new foreign SVAR model, for which the identification scheme has changed significantly with only four rather than six structural shocks being separately identified.

To assess whether the modifications made help the updated and re-estimated model version 1.4 to fit the data, a direct comparison with the previous version 1.2.3 is not feasible due to the use of the additional observed non-oil import-price variable and the changes to the measurement of oil prices and competitors' export prices. However, version 1.4 can be compared with a variant which incorporates the new direct oil price propagation channel on the consumer side and uses the same observed data as model version 1.4, but where the foreign SVAR from the previous model version is first re-estimated and then employed in the re-estimation of the parameters of the extended structural model. For these two cases, the Laplace approximation of the log marginal likelihood is considerably larger when the new foreign SVAR is used (with a difference of nearly 700 log units), clearly supporting the use of the new foreign SVAR model.

4 Model properties

4.1 Model and sample moments

Table 4 reports the population means and standard deviations of the observed variables based on the updated and re-estimated model and its posterior mode estimates of the parameters, along with data-based estimates of these moments for the sample period 1985–2019. The results suggest that the model-based means are broadly in line with the data-based means. In particular, for the real variables the data-based means are close to the model-based means, with the exception of long-term growth expectations. For the inflation measures, the data-based means are moderately larger than the model-based means, with the exception of the two import price deflators. Turning to the standard deviations, the data-based estimates are typically lower than the model-based values, with exceptions being

previous model version (0.35), for which no system prior was imposed on the reduced-form slope parameter.

employment, the short-term nominal interest rate and the 10-year government bond yield.[10] However, compared with the previous model version (see Coenen et al., 2022, Table 7) the model-based standard deviations are overall closer to the data-based estimates, especially for the real effective exchange rate.

4.2 Impulse response functions

Shock to the short-term nominal interest rate

In Figure 3 we depict the impulse responses of selected domestic variables to a shock to the short-term nominal interest rate and compare the responses obtained for the extended version of the model to those for the previous one. The results show that the changes made to the model have a very small impact on the transmission of monetary policy shocks. As in the previous version of the model, a monetary policy shock leads to a hump-shaped decline in domestic demand, with a larger impact on investment. The lower demand leads to lower employment and, thus, real wages fall too. The decline in wages contributes to the decline in firms' marginal costs which puts downward pressure on domestic prices and, by extension, on consumer prices. Overall, in the extended version of the model, prices react somewhat more strongly to the monetary policy shock, and real wages decline by less.

Shock to the price of oil

Figure 4 compares the impulse responses to a 10% oil price shock. We focus on an oil price shock to ascertain the new channels in the extended version of the model that give rise to an enhanced propagation of oil price changes. In response to the oil price shock, aggregate import prices rise significantly more in the extended model, masking an immediate sharp increase in oil import prices and a more muted increase in non-oil import prices on account of a depreciation of the euro exchange rate. Overall, the substantial increase in import prices translates into higher consumer price inflation in the extended model, with a response pattern which is more in line with empirical benchmarks.[11] By contrast, in the previous version of the model, the oil price shock translated into a very small response of

[10] Because of the shorter sample, the comparison of the moments for the composite long-term lending rate is not meaningful.

[11] In particular, the response pattern of consumer price inflation is broadly comparable to that obtained from applying the Basic Model Elasticities (BMEs), which represent a widely used benchmark in the Eurosystem/ECB staff projection exercises (see ECB, 2016, "A guide to the Eurosystem/ECB staff macroeconomic projection exercises").

consumer prices.[12] The increase in consumer price inflation leads to a decline in real wages, reducing real income. Thus, domestic demand declines. While the response of investment is similar, the fall in private consumption is more substantial in the extended model and translates into a larger decrease in real GDP. The GDP decrease in turn leads to a reduction in labour demand, implying in equilibrium lower employment and a further decline in real wages. The increase in import prices also implies a reduction in imports, although it is more muted than in the original version of the model reflecting the low elasticity of substitution of oil in aggregate consumption. Overall, the extended version of the model yields stronger nominal and real responses to an oil price shock, which are arguably more realistic.

4.3 Historical decompositions

As the last element in our documentation of the model properties, we present the contributions of the shocks obtained from the updated and re-estimated version of NAWM II to the observed fluctuations in per capita real GDP growth and consumer price inflation. We focus on the historical period from 2000 to 2019 and bundle the shocks into seven groups: technology shocks, demand shocks, mark-up shocks, financial shocks, foreign shocks and perception shocks, plus a monetary policy shock, but we also show the breakdown of the contribution from the foreign shock group into the contributions of the individual structural shocks and the group of idiosyncratic shocks from the SVAR model. As will be discussed below, the financial shocks carry both demand and supply features, albeit at different horizons of their propagation to the economy.

Real GDP growth

The historical decomposition of per capita real GDP growth depicted in Figure 5 shows that the years of strong growth both in the early part of the period considered and during the run-up to the financial crisis of 2008-09 is explained to a considerable extent by positive contributions of financial shocks. These positive contributions reflect the very favourable financing conditions at the time, fostering domestic demand. In both sub-periods, the financial shocks more than offset the persistently negative contribution from adverse technology shocks, on the back of a positive contribution from (wage) mark-up shocks reflecting a

[12] It should be emphasised, however, that in the previous version of the model, the shocks were identified using a Cholesky decomposition. Therefore, it is difficult to assign a clear economic interpretation to the shock, which may capture movements in oil prices not strictly related to oil markets.

protracted period of wage moderation in the pre-crisis years.

The sharp and drawn-out decline in real GDP growth caused by the unfolding of the financial crisis in 2008 is attributed in part to a negative contribution of foreign shocks, capturing the precipitous fall in world trade and the implied contraction in euro area foreign demand, and an even larger negative contribution from financial shocks. The latter contribution arguably reflects the sudden freezing of financial markets which gave rise to a surge in risk premia. In addition, a notable negative contribution from monetary policy shocks is identified, despite the fact that the ECB, like other major central banks around the world, rapidly reduced its key interest rates to historically low levels in order to bolster domestic demand and stabilise financial markets. Finally, the prolonged slump in GDP growth is accompanied by a lasting weakness in investment and a slowing of trend productivity growth. These developments are explained by an increasing negative contribution of technology shocks.

The subsequent recovery in GDP growth in 2009 and 2010 is explained by a marked turnaround in the negative contributions of the foreign and financial shocks, reflecting the stabilisation of both the world economy and financial markets. In addition, the decomposition suggests that monetary policy shocks forcefully underpinned the recovery reflecting the fact that the ECB kept its key interest rates at low levels despite of the strong recovery.[13] The setback in the recovery in the course of late 2010 and 2011 – owing to the re-intensification of the financial crisis on the back of elevated tensions in euro-area sovereign debt markets – is explained to a large extent by the waning of the previously positive contribution from financial shocks (tantamount to a renewed surge in risk premia in long-term interest rates), in conjunction with a continuing negative contribution from technology shocks against the backdrop of a persistent weakness of investment and a resumed slowing of trend productivity growth.

The lingering effects of the financial and sovereign debt crises continued to weigh adversely on GDP growth until 2020, as reflected in the lasting negative contribution of

[13] When interpreting the findings regarding the role of monetary policy shocks during the crisis and the subsequent recovery due account should be given to the fact that the ECB also swiftly implemented non-standard monetary policy measures, including the provision of unlimited liquidity to the banking system, in order to sustain financial intermediation and to maintain the availability of credit to the private sector. The effects of these non-standard measures are not directly captured by the model-based contribution analysis, but only indirectly and, arguably, via positive contributions of the financial shocks.

technology shocks. From 2014 onwards, this contribution is largely offset by a persistent contribution of financial shocks arguably capturing the ECB's adoption of new non-standard measures, including interest-rate forward guidance and large-scale asset purchases. The latter lowered the level of long-term interest rates by compressing term premia, over and above the gradual lowering of the ECB's key policy rates towards the effective lower bound and its forward guidance, which is in part explained by a positive contribution of monetary policy shocks. In addition, GDP growth was upheld by an initially positive contribution of foreign shocks, reflecting the stronger economic performance abroad, and an increasingly important positive contribution from mark-up shocks, reflecting subdued price and wage pressures during the ensuing low inflation episode.

Inflation

The historical decomposition of annual consumer price inflation depicted in Figure 6 suggests that past inflation developments were driven by a combination of cyclical and structural, i.e. highly persistent, forces. First, amongst the persistent forces, the sequence of negative technology shocks identified over most of the period under consideration results in persistent upward pressures on inflation via their adverse impact on firms' marginal costs. Yet the implied inflationary momentum is offset, if not overcompensated, by a lasting negative contribution from favourable financial shocks. While the latter have a positive impact on demand in the shorter term and hence ought to positively affect inflation, these shocks, notably the highly persistent discount-rate shock capturing the secular downward trend in real interest rates, stimulate investment and thus enhance the economy's productive capacity. It is via a decline in the rental cost of capital that the enhanced productive capacity results in lower marginal costs and puts lasting downward pressure on inflation over the medium to longer term. In a similar vein, albeit less lasting, negative mark-up shocks, notably wage mark-up shocks, have weighed negatively on inflation, especially during the period of wage moderation prior to the financial crisis and during most of the low-inflation episode after the financial and sovereign debt crises.

And second, mirroring their cyclical effects on real GDP growth as described above, demand and monetary policy shocks tend to contribute to the cyclical fluctuations in inflation, albeit with limited effects as the key price and wage Phillips curves of the model are rather flat, reflecting the high degrees of nominal and regal rigidities in the price and wage-setting behaviour of firms and households.

Finally, foreign shocks, and amongst those most notably oil price shocks, explain a significant part of the fluctuations in inflation. In particular, oil price shocks associated with the surges in oil prices during the period prior to the financial crisis and following the recovery from it can explain the episodes of heightened inflation over the period considered. Likewise, the disinflation episodes in the intermediate aftermath of the financial crisis, but also in the course of 2015, can be attributed in large part to foreign shocks and, especially, to oil price shocks capturing sharp corrections in oil prices.

Table 1: Sign restrictions for identification of the SVAR shocks

	Supply	Demand	Mon. policy	Oil price
Foreign prices	+	+	−	+
Foreign demand	−	+	−	−
Foreign short-term rate	?	+	+	?
Foreign long-term rate	?	+	+	?
Real price of oil	−	−	+	+
Competitors' export prices	+	+	−	+

Note: A '+' indicates that the variable is supposed to increase after the shock. A '−' indicates that the variable will decrease after the shock. Finally, a '?' indicates that no sign restriction is imposed on that variable for the corresponding shock. The restrictions apply to the contemporaneous impacts of the different shocks.

Table 2: Prior distributions and posterior mode estimates of the structural parameters

Parameter	Description	Prior distribution	Posterior mode	
			v1.4	v1.2.3
A. Preferences				
κ	Habit formation	beta(0.75,0.10)	0.63	0.69
B. Wage and price setting				
ξ_W	Calvo scheme: wages	beta(0.75,0.0375)	0.79	0.77
χ_W	Indexation to inflation: wages	beta(0.75,0.10)	0.32	0.36
$\tilde{\chi}_W$	Indexation to productivity: wages	beta(0.75,0.10)	0.78	0.67
ξ_H	Calvo scheme: domestic prices	beta(0.75,0.0375)	0.81	0.83
χ_H	Indexation: domestic prices	beta(0.75,0.10)	0.33	0.21
ξ_X	Calvo scheme: export prices	beta(0.75,0.0375)	0.77	0.80
χ_X	Indexation: export prices	beta(0.75,0.10)	0.35	0.28
ξ^*	Calvo scheme: import prices	beta(0.75,0.0375)	—	0.58
ξ^*_O	Calvo scheme: import prices	beta(0.25,0.0375)	0.06	—
ξ^*_{XO}	Calvo scheme: import prices	beta(0.75,0.0375)	0.69	—
χ^*	Indexation: import prices	beta(0.75,0.10)	—	0.27
χ^*_O	Indexation: import prices	beta(0.25,0.10)	0.18	—
χ^*_{XO}	Indexation: import prices	beta(0.75,0.10)	0.34	—
o^*	Oil import price weight	beta(0.15,0.05)	0.13	0.26
C. Wage and domestic price Phillips curves				
$100 \cdot sl^W$	Slope parameter	gamma(0.50,0.10)	0.29	—
$100 \cdot sl^H$	Slope parameter	gamma(1.00,0.15)	0.74	0.68
D. Final and intermediate-good production				
μ_C	Subst. elasticity: consumption	gamma(1.50,0.25)	—	2.99
μ_C	Subst. elasticity: consumption	gamma(0.40,0.10)	0.40	—
μ_{CXO}	Subst. elasticity: consumption	gamma(1.50,0.25)	2.12	—
μ_I	Subst. elasticity: investment	gamma(1.50,0.25)	1.39	1.33
μ_X	Subst. elasticity: exports	gamma(1.50,0.25)	0.86	0.78
μ^*	Price elasticity: exports	gamma(1.50,0.25)	1.20	1.41
α	Capital share	beta(0.36,0.10)	0.35	0.31
E. Wholesale and retail banks				
ω_L	Absconding of domestic bonds	gamma(0.72,0.05)	0.72	0.71
$\tilde{\omega}^*_L$	Absconding of foreign bonds	gamma(1.00,0.05)	1.00	0.98
ξ_I	Calvo scheme: price of loans	beta(0.75,0.0375)	0.71	0.73

Note: This table provides information on the marginal prior distributions and the posterior mode estimates of the structural parameters for versions 1.4 and 1.2.3 of NAWM II. The prior distributions are characterised by the parameters determining their respective means and variances. The posterior mode columns show the estimates obtained by numerically maximising the posterior distribution.

Table 2: Prior distributions and posterior mode estimates of the structural parameters (cont'd)

Parameter	Description	Prior distribution	Posterior mode	
			v1.4	v1.2.3
F. Adjustment costs				
γ_I	Investment	gamma(6.00,1.50)	6.19	9.30
$\gamma_{u,2}$	Capital utilisation	gamma(1.00,0.25)	0.72	0.82
γ_{IM^C}	Import content: consumption	gamma(2.50,1.00)	—	6.80
$\gamma_{IM^{CO}}$	Import content: consumption	gamma(5.00,1.00)	3.81	—
$\gamma_{IM^{CXO}}$	Import content: consumption	gamma(2.50,1.00)	5.75	—
γ_{IM^I}	Import content: investment	gamma(2.50,1.00)	0.73	0.61
γ^*	Export market share	gamma(2.50,1.00)	1.65	2.76
γ_L^h	Portfolio: households	gamma(0.01,0.0025)	0.009	0.008
γ_L^*	Portfolio: wholesale banks	gamma(0.01,0.0025)	0.004	0.002
G. Interest-rate rule				
ϕ_R	Interest-rate smoothing	beta(0.90,0.05)	0.93	0.93
ϕ_Π	Response to inflation	gamma(1.70,0.575)	2.52	2.93
$\phi_{\Delta\Pi}$	Response to change in inflation	gamma(0.30,0.125)	0.04	0.04
ϕ_Y	Response to output gap	gamma(0.14,0.05)	0.04	0.03
$\phi_{\Delta Y}$	Response to change in output gap	gamma(0.07,0.025)	0.12	0.09
H. Employment (bridge) equation				
ξ_E	Calvo-style weighing scheme	beta(0.50,0.15)	0.85	0.85
I. Perception updating equations				
$\varpi_{\bar{\Pi}^p}$	Sens. of perc. inflation objective	gamma(0.10,0.25)	0.05	0.05
$\varpi_{g_Y^p}$	Sens. of perc. trend growth rate	gamma(0.10,0.25)	0.06	0.06

Note: See above.

Table 3: Prior distributions and posterior mode estimates of the parameters of the shock processes

Parameter	Description	Prior distribution	Posterior mode	
			v1.4	v1.2.3
A. Autoregressive parameters				
ρ_{RP}	Domestic risk premium shock	beta(0.75,0.10)	0.86	0.91
ρ_{RP^*}	External risk premium shock	beta(0.75,0.10)	0.94	0.99
$\rho_{g_z^p}$	Perm. techn. shock: pers. comp.	beta(0.75,0.10)	0.97	0.93
ρ_{ε}	Transitory technology shock	beta(0.75,0.10)	0.83	0.88
ρ_I	Investment-specific techn. shock	beta(0.75,0.10)	0.91	0.91
ρ_{φ^W}	Wage mark-up shock	beta(0.50,0.05)	0.67	0.72
ρ_{φ^H}	Domestic price mark-up shock	beta(0.50,0.05)	0.53	0.59
ρ_{φ^X}	Export price mark-up shock	beta(0.50,0.05)	0.44	0.40
ρ_{φ^*}	Import price mark-up shock	beta(0.50,0.05)	—	0.49
$\rho_{\varphi^*_O}$	Import price mark-up shock	beta(0.50,0.05)	0.47	—
$\rho_{\varphi^*_{XO}}$	Import price mark-up shock	beta(0.50,0.05)	0.46	—
ρ_{IM}	Import demand shock	beta(0.75,0.10)	0.84	0.91
ρ_{ν^*}	Export preference shock	beta(0.75,0.10)	0.89	0.90
ρ_{υ}	Discount rate shock	beta(0.75,0.10)	0.99	0.99
ρ_{φ^I}	Loan rate mark-down shock	beta(0.75,0.05)	0.78	0.75
B. Scaling parameters				
σ_{RP}	Domestic risk premium shock	invgamma(0.122474,2)	0.23	0.20
σ_{RP^*}	External risk premium shock	invgamma(0.122474,2)	0.24	0.12
$\sigma_{g_z^p}$	Perm. techn. shock: pers. comp.	invgamma(0.122474,2)	0.04	0.07
σ_{ε}	Transitory technology shock	invgamma(0.122474,2)	1.17	1.18
σ_I	Investment-specific techn. shock	invgamma(0.122474,2)	0.32	0.26
σ_{φ^W}	Wage mark-up shock	invgamma(0.122474,2)	0.09	0.08
σ_{φ^H}	Domestic price mark-up shock	invgamma(0.122474,2)	0.11	0.13
σ_{φ^X}	Export price mark-up shock	invgamma(0.122474,2)	0.91	0.77
σ_{φ^*}	Import price mark-up shock	invgamma(0.122474,2)	—	1.26
$\sigma_{\varphi^*_O}$	Import price mark-up shock	invgamma(0.122474,2)	13.31	—
$\sigma_{\varphi^*_{XO}}$	Import price mark-up shock	invgamma(0.122474,2)	0.99	—
σ_{IM}	Import demand shock	invgamma(0.122474,2)	6.63	6.52
σ_{ν^*}	Export preference shock	invgamma(0.122474,2)	6.58	9.75
σ_R	Interest rate shock	invgamma(0.122474,2)	0.10	0.10
σ_{υ}	Discount rate shock	invgamma(0.122474,2)	0.04	0.05
σ_{φ^I}	Loan rate mark-down shock	invgamma(0.122474,2)	0.92	0.97
$\sigma_{\bar{\Pi}^p}$	Perc. inflation objective shock	invgamma(0.122474,2)	0.02	0.02
$\sigma_{g_Y^p}$	Perc. trend growth rate shock	invgamma(0.122474,2)	0.02	0.02
C. Signal-to-noise ratio				
$\sigma^2_{g_z^p}/\sigma^2_{g_z^{tr}}$	Permanent technology shock	gamma(0.05,0.005)	0.06	0.06

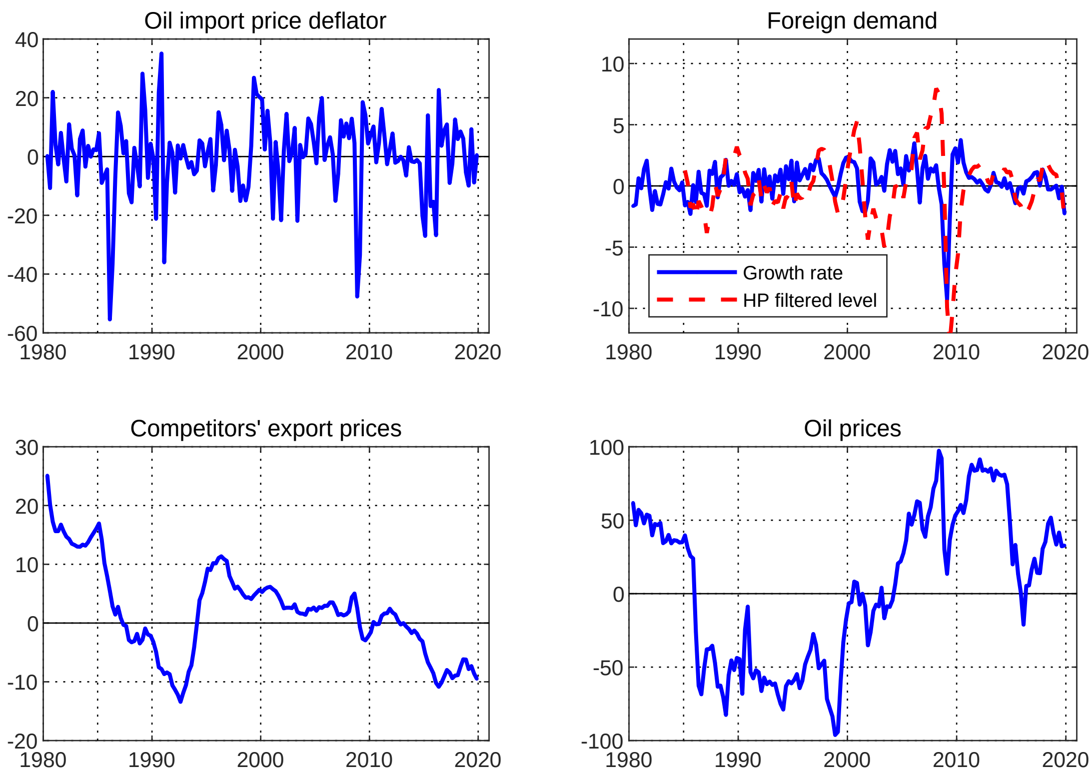
Note: See Table 2. The inverse gamma prior distributions are parameterised in terms of their mode and their degrees of freedom.

Table 4: Means and standard deviations of the observed variables

Variable	Mean		Standard deviation	
	NAWM II	Data	NAWM II	Data
Real GDP	0.375	0.40	0.73	0.57
Consumption	0.375	0.37	0.82	0.47
Investment	0.375	0.41	3.06	1.50
Gov't consumption	0.00	0.00	0.82	0.87
Exports	0.375	0.40	4.04	2.32
Imports	0.375	0.40	2.84	2.06
GDP defl. inflation	0.50	0.55	0.55	0.33
Consumption defl. inflation	0.50	0.52	0.56	0.37
Import defl. inflation	0.50	0.01	2.81	2.48
Oil import defl. inflation	0.50	0.05	13.26	13.71
Employment	0.00	0.00	1.90	2.85
Wage inflation	0.80	0.77	0.66	0.50
Short-term nom. interest rate	3.25	4.26	2.24	3.67
Real effective exchange rate	0.00	0.00	13.42	12.28
10-year gov't bond yield	4.66	4.51	1.47	2.46
Comp. long-term lending rate	5.41	3.45	1.48	1.15
Long-term inflation expect's	2.00	1.89	1.05	0.07
Long-term growth expect's	1.50	1.97	0.53	0.34
Output gap	0.00	−0.16	6.44	1.81

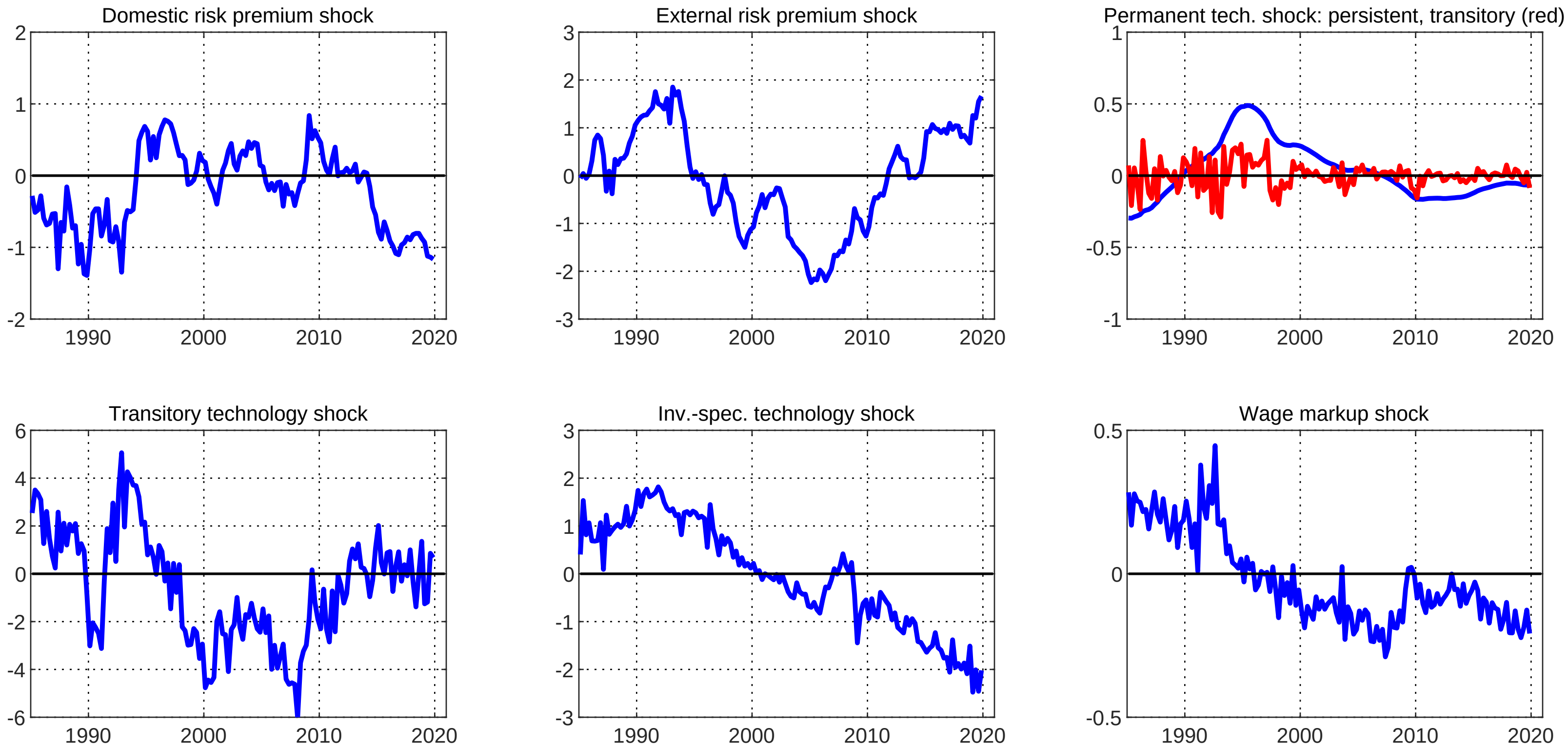
Note: This table reports population means and standard deviations at the posterior mode estimates of NAWM II, version 1.4, for the observed variables used in its Bayesian estimation, along with the corresponding sample moments based on the data covering the period 1985Q1–2019Q4.

Figure 1: The new transformed data



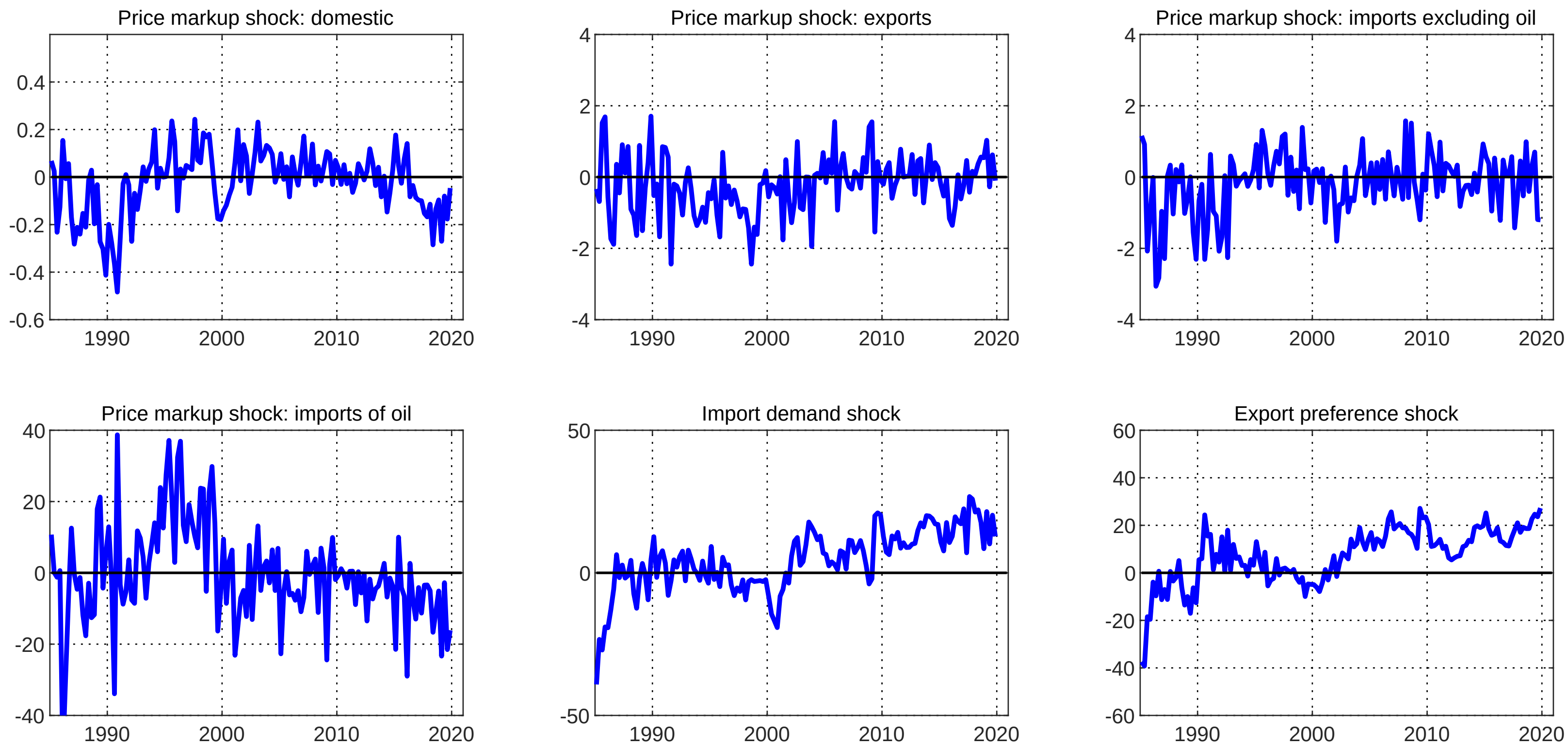
Note: This figure shows the new and modified time series used in the estimation of NAWM II, version 1.4.

Figure 2: Smoothed estimates of shocks



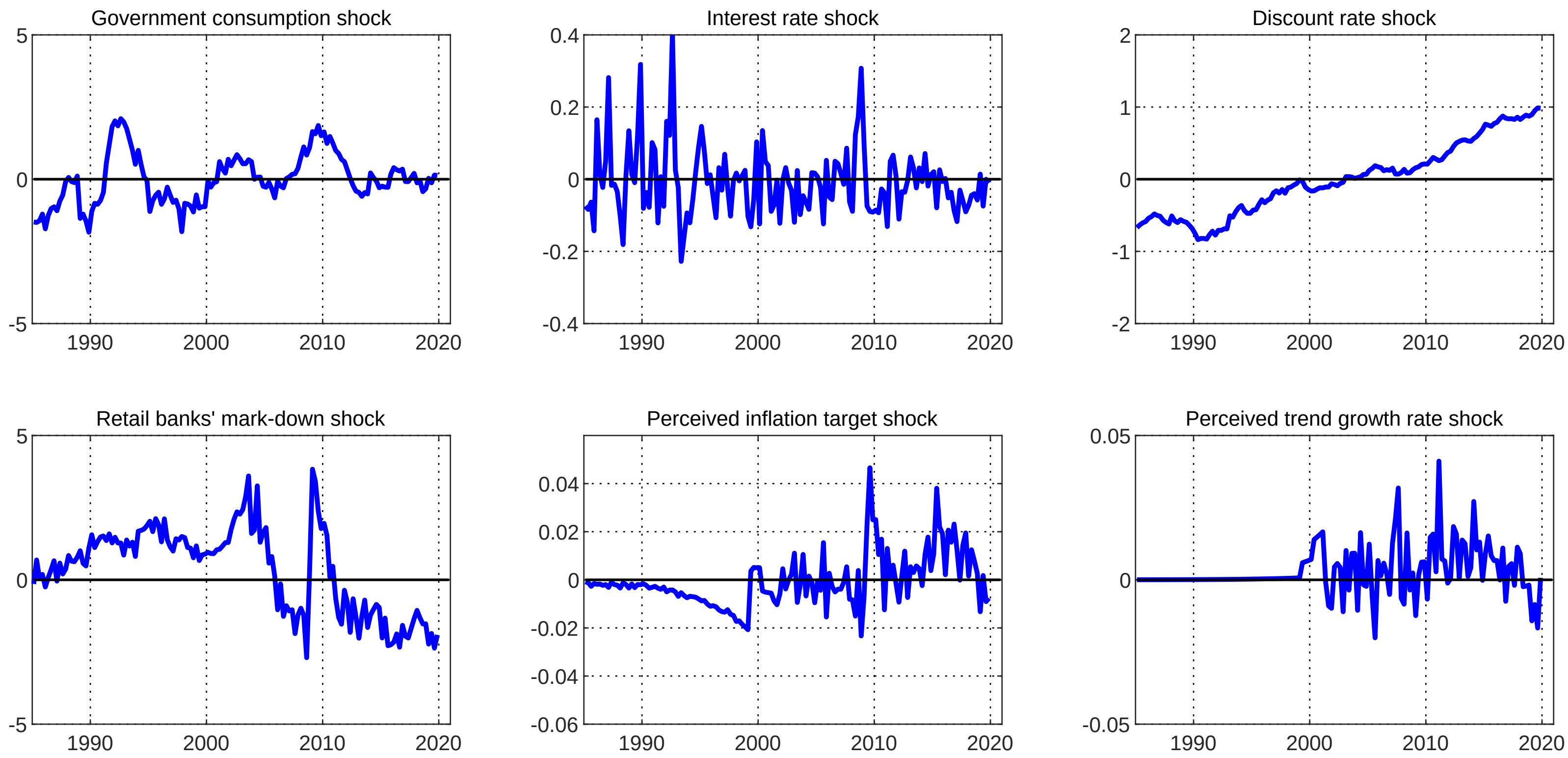
Note: This figure shows the smoothed estimates of the structural shocks of NAWM II, version 1.4, conditional on the posterior mode estimates of its parameters.

Figure 2: Smoothed estimates of shocks (continued)



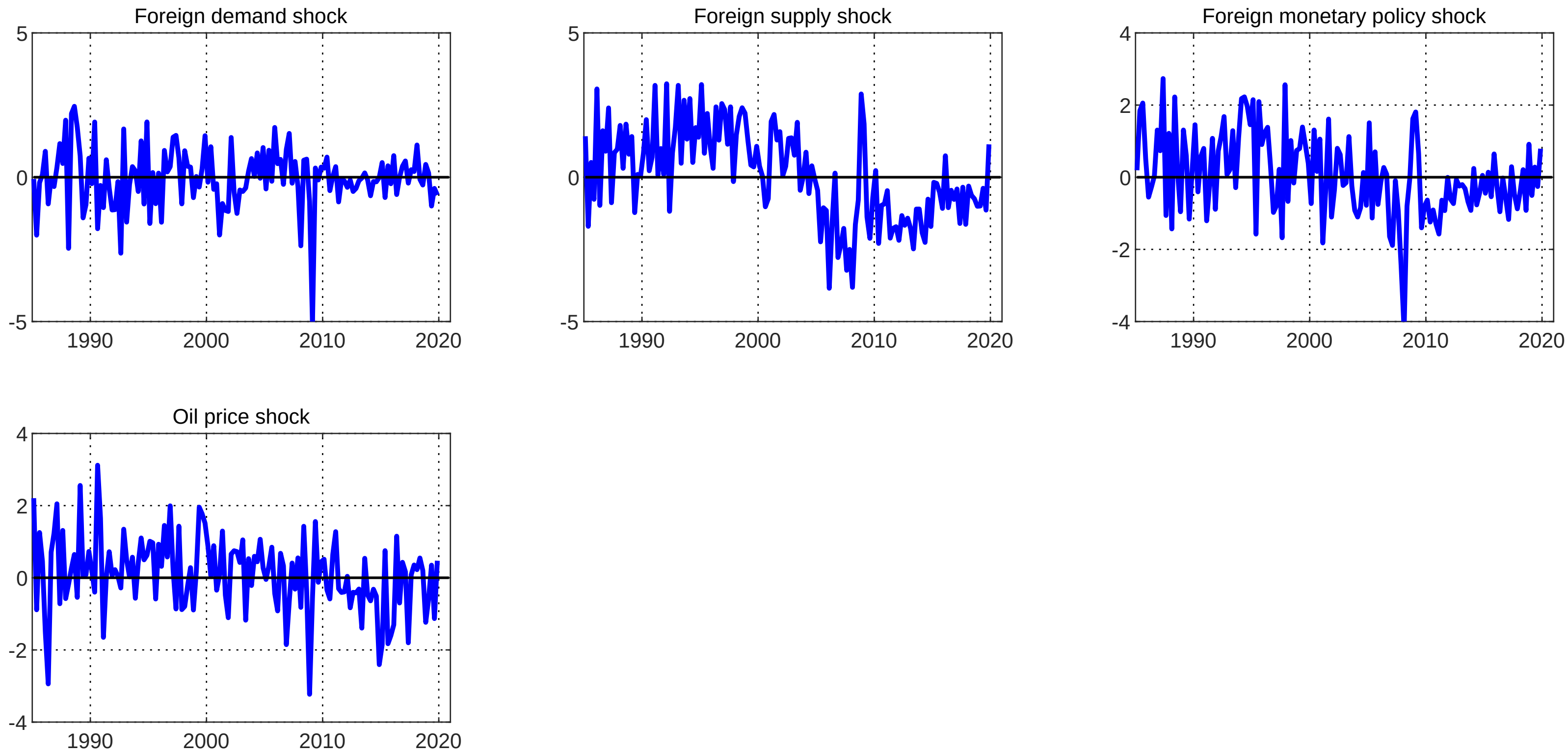
Note: See above.

Figure 2: Smoothed estimates of shocks (continued)



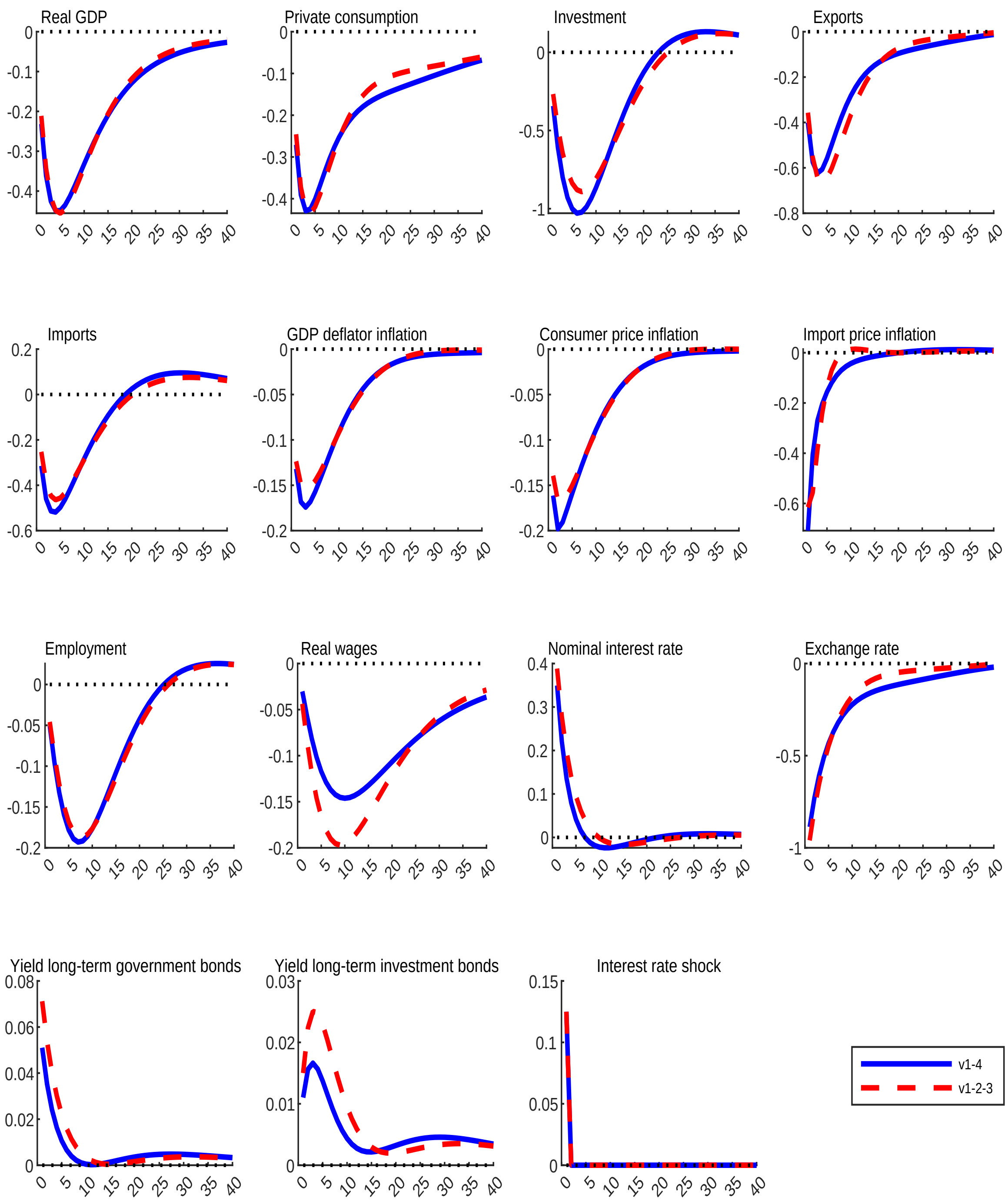
Note: See above.

Figure 2: Smoothed estimates of shocks (continued)



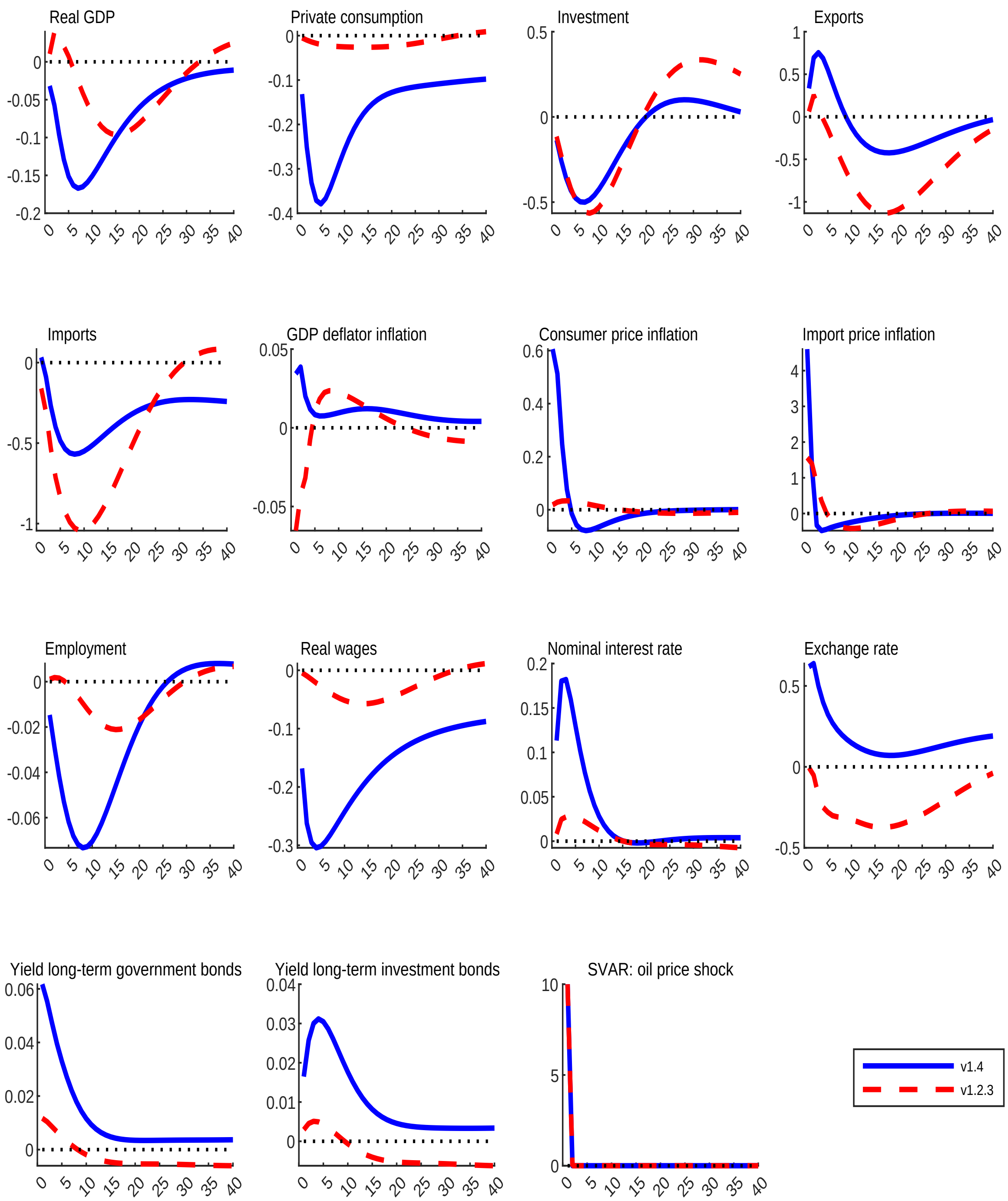
Note: See above.

Figure 3: Impulse responses to an interest rate shock



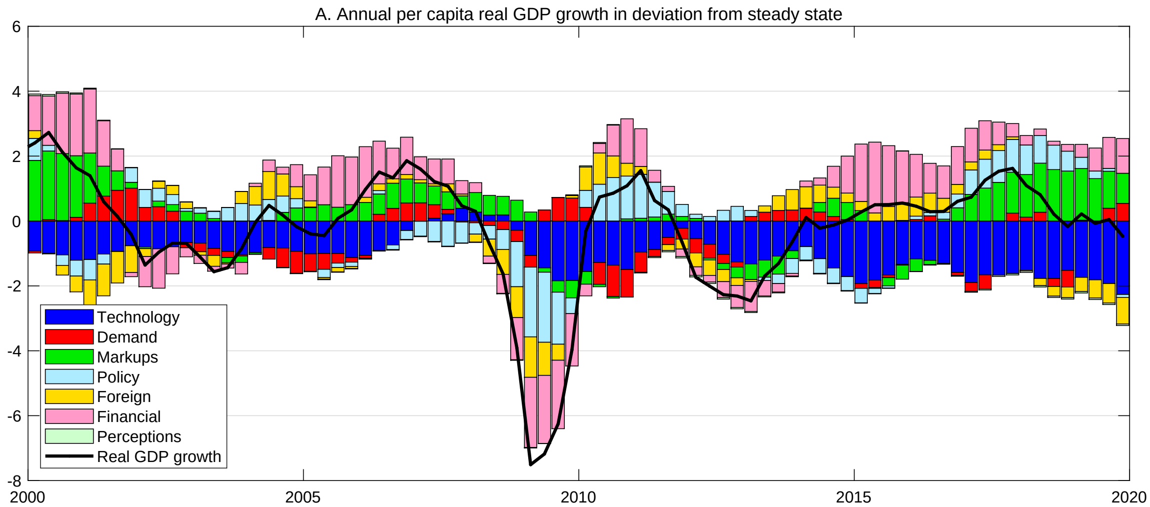
Note: For versions 1.4 and 1.2.3 of NAWM II, this figure depicts the impulse responses of selected domestic variables to an interest rate shock equal to 50 basis points in annualised terms. All impulse responses are reported as percentage deviations from the non-stochastic balanced growth path, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations.

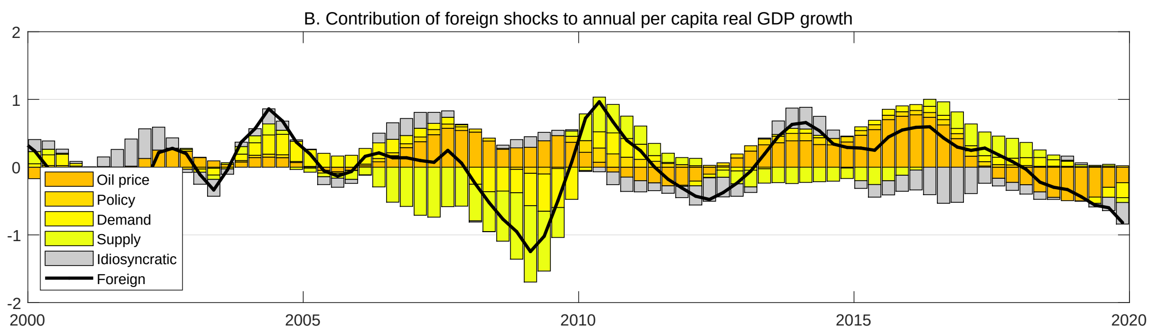
Figure 4: Impulse responses to an oil price shock



Note: For versions 1.4 and 1.2.3 of NAWM II, this figure depicts the impulse responses of selected domestic variables to a 10% oil price shock. All impulse responses are reported as percentage deviations from the non-stochastic balanced growth path, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations.

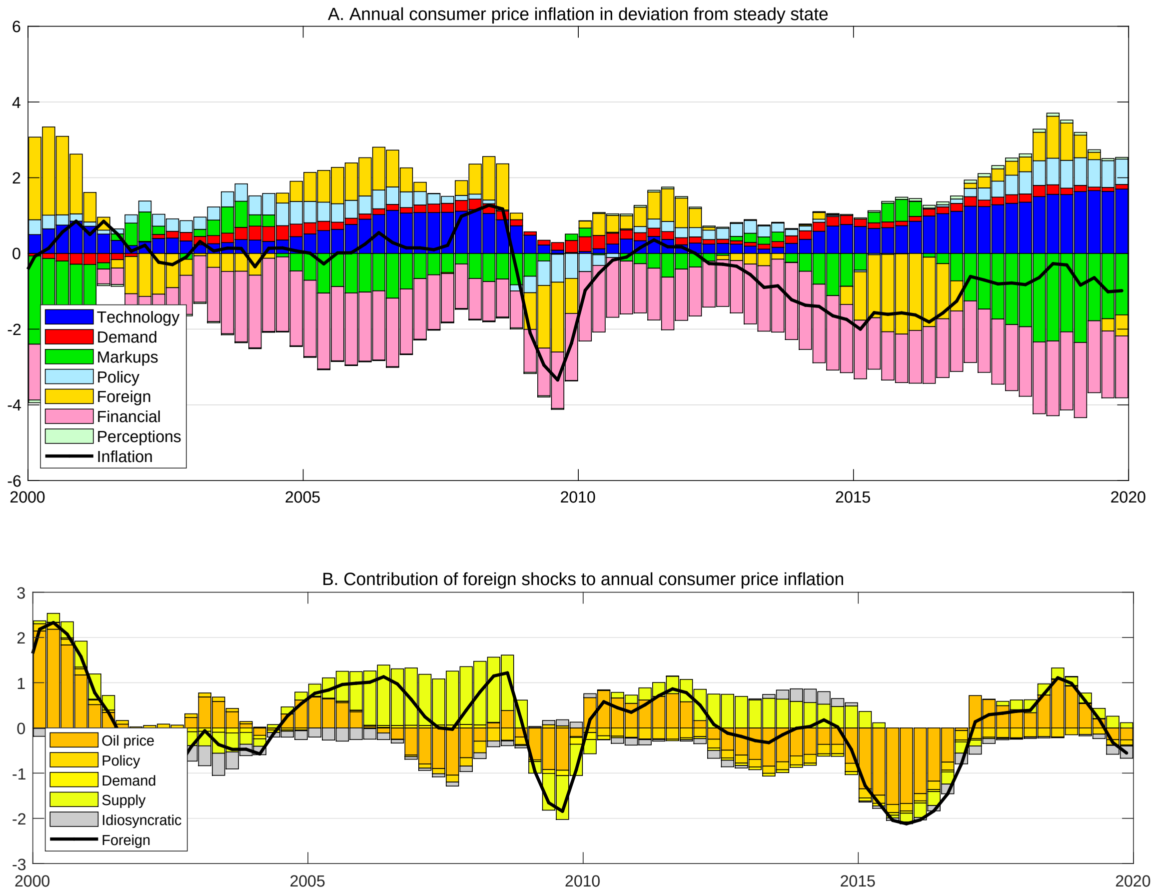
Figure 5: Historical decomposition of real GDP growth





Note: The upper panel in this figure shows the decomposition of annual per-capita real GDP growth into the contributions of the structural shocks of NAWM II, version 1.4, over the period 2000 to 2019. The shocks are bundled into seven groups: technology, demand, mark-up, foreign, financial and perception shocks, plus a monetary policy shock. The decomposition is computed in deviation from the model-implied steady-state growth rate of 1.5% using the posterior mode estimates of the model's estimated parameters.The lower panel shows the contributions of the individual foreign shocks to annual per capita real GDP growth.

Figure 6: Historical decomposition of consumer price inflation



Note: The upper panel in this figure shows the decomposition of annual consumer price inflation into the contributions of the structural shocks of NAWM II, version 1.4, over the period 2000 to 2019. The shocks are bundled into seven groups: technology, demand, mark-up, foreign, financial and perception shocks, plus a monetary policy shock. The decomposition is computed in deviation from the model-implied steady-state inflation rate of 2% using the posterior mode estimates of the model's estimated parameters.The lower panel shows the contributions of the individual foreign shocks to annual consumer price inflation.