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Marco Gross, Jérôme Henry, Willi Semmler

## Destabilizing effects of bank overleveraging on real activity - an analysis based on a threshold MCS-GVAR

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**Abstract**

We investigate the consequences of overleveraging and the potential for destabilizing effects from financial- and real-sector interactions. In a theoretical framework, we model overleveraging and indicate how a highly leveraged banking system can lead to unstable dynamics and downward spirals. Inspired by Brunnermeier and Sannikov (2014) and Stein (2012), we empirically measure the deviation-from-optimal-leverage for 40 large EU banks. We then use this measure to condition the joint dynamics of credit flows and macroeconomic activity in a large-scale regime change model: A Threshold Mixed-Cross-Section Global Vector Autoregressive (T-MCS-GVAR) model. The regime-switching component of the model aims to make the relationship between credit and real activity dependent on the extent to which the banking system is overleveraged. We find significant nonlinearities as a function of overleverage. When leverage is standing above its equilibrium level, the effect of a deleveraging shocks on credit supply and economic activity are visibly more detrimental than at times of underleveraging.

Keywords: Macro-financial linkages, overleveraging, credit supply

JEL classification: E2, E6, C13, G6

# Non-technical summary

Much recent research point to the excessive leveraging of the banking sector as being the essential driving force for macroeconomic instability. We develop a theoretical model to demonstrate how a highly leveraged banking system can lead to unstable dynamics and downward spirals. Then, inspired by a method by Brunnermeier and Sannikov (2014) and Stein (2012) we implement an empirical measure of overleveraging for a large sample of EU banks. Subsequently we include the overleveraging measure in a large scale macro model, by aggregating the 40 banks' individual measures of overleveraging into 14 banking systems and use the overleveraging variable as a transition variable in a Threshold Mixed-Cross-Section GVAR model for 28 countries and 14 banking systems of the EU.

Our empirical analysis results in three main conclusions. First, the reason for why macroeconomic responses to bank capital shocks are likely significantly stronger under an overleveraging regime is that the same percentage point capital ratio shock translates into a stronger asset side reaction (which is particularly visible under a scenario whereby banks choose to shrink instead of raising capital to achieve higher capital ratios) than under an initial low-leverage (i.e. high capital ratio) regime. This effect is very much mechanic, though quite essential.

Second, our estimates suggest that GDP to credit long-run response ratios are higher under a bank overleveraging regime. It is an effect that arises in addition to the shock amplification effect (previous point), whereby credit and GDP tend to co-move more strongly, hence rendering economic activity more susceptible to changes in bank credit supply under the overleveraging regime.

Third, cross-border spillover effects appear to be more pronounced when capital ratio shocks hit the banking system during a period of overleveraging. This finding reflects the fact that bank credit supply does not only exert its state-dependent effect on the domestic economy but implies cross-border effects through the cross-border supply of credit.

From a policy perspective, by distinguishing explicitly between *contractionary* and *expansionary* deleveraging in our simulations, we aim to raise the awareness that it matters how banks go about moving to higher capital ratios and that macroprudential policy makers ought to take into account and give concrete guidance depending on the context, i.e. on prevailing macroeconomic conditions and the purpose of the instrument. When imposing macroprudential capital buffer requirements during times of weak economic activity, a capital shortfall that banks may face should be filled rather by raising capital and not by further shrinking banks' balance sheet size. Compressing asset growth would imply the risk of dragging economic activity further as a result of falling bank credit supply and thus render recessions deeper. During times of expansion, on the other hand, shrinking the asset side rather than raising equity might be desired for achieving the macroprudential objective of dampening the financial cycle (e.g. using a countercyclical capital buffer).

# 1 Introduction

The banking sector has, in particular since the outbreak of the global financial crisis, gained in prominence for what concerns the public perception and political as well as academic interest in macro-financial linkages, i.e. its role in causing and amplifying economic recessions around the world. Numerous studies confirm the understanding that the US financial crisis of the years 2007-09 was caused by excessive leveraging of the US banking sector, its exposure to the US real estate sector, and the highly nonlinear amplification effects that arose once the banking sector became vulnerable and unstable after the arrival of adverse shocks. The regulatory proposals of Basel III after the years 2009-10 were concentrated on the excessive leveraging of the banking sector, pointing to the fact that overleveraging renders the banking and financial sector fragile.

Credit cycles are a common feature of financial systems and tend to positively correlate with the business cycle, reflecting fluctuations in borrowers' demand for, and need of, financing.[1] Cycles in credit developments and thereby implicitly in financial sector leverage are exacerbated by the inherent pro-cyclical behavior of financial intermediaries.[2] Experience from past financial crises indicates that the depth and length of crises tend to be stronger when they were preceded by credit booms (Reinhart and Rogoff (2009), Laeven and Valencia (2012)). This insight has led financial regulators around the world to consider counter-cyclical policy measures to help alleviate financial cycle fluctuations (Drehmann et al. (2011)).

There are numerous studies which address the mechanisms of how excessive leveraging and fragility of the banking sector can lead to adverse macroeconomic feedback effects. Brunnermeier and Sannikov (2014) focus, in a stylized manner, on financial experts representing financial intermediaries that highly leverage. In their view it is a shock to asset prices which creates a vicious cycle through the balance sheets of the banks, contagion effects and macro feedback loops. In Mittnik and Semmler (2013), the vulnerability of banks and the system's instability is seen to be caused by the limited liabilities of decision makers in banks, improper incentive systems and lack of constraints imposed on financial institutions, that allow for unrestricted growth of capital assets (through borrowing). In Stein (2012) the destabilizing mechanism also results from a linkage of asset prices and borrowing. When the interlinkages of asset prices and borrowing allow the bank to enjoy capital gains beside the normal returns, they start to become overleveraged.[3]

We introduce our own measure of overleveraging (following in spirit Brunnermeier and Sannikov (2014) and Stein (2012)), defining overleveraging as a positive gap between a borrower's actual debt levels and its debt capacity. Debt capacity can alternatively be referred to as sustainable debt, or *optimal debt*. As to the empirical estimates of the measurement of excessive debt for the EU countries, we follow the method developed by Schleer et al. (2014). In a large scale regime switching Global Vector Autoregressive (GVAR) model we then evaluate the overleveraging hypothesis for a regime of high and low leveraging of the banking sector. We assess whether regimes of overleveraging of the banking sector are accompanied by more severe credit and output contractions than periods of underleveraging, both in response to otherwise equal supply shocks to credit.

A useful entry point to the GVAR literature is a recent survey paper by Chudik and Pesaran (2014), and the contributions by Pesaran et al. (2004), Pesaran and Smith (2006), and Dees et al. (2007). Our model uses the Mixed-Cross Section variant of the GVAR that was developed by Gross and Kok (2013) and applied to banks' balance sheets in Gross et al. (2016). It allows combining different cross-

[1] See Borio et al. (2001) and Brunnermeier and Shin (2011). See also Hiebert et al. (2014) for a recent analysis of the co-movements of financial and business cycles in the euro area.

[2] See Kiyotaki and Moore (1997), Bernanke et al. (1999), Allen and Gale (2004), Rajan (2006), Geanakoplos (2009), Adrian and Shin (2010), Schularick and Taylor (2012) and Claessens et al. (2012).

[3] There are also a number of DSGE models which point to the possible destabilizing role of financial institutions. See for example Christiano and Ikeda (2013) and Gerali et al. (2010).

section types (countries with banks or banking systems, central banks, etc.) in a GVAR variant that is referred to as Mixed-Cross-Section (MCS). The model framework that we use in this paper extends the MCS-GVAR by yet another feature—a regime conditionality, in our application upon the degree of overleveraging of a banking system. The global solution of the model becomes *regime constellation dependent*.[4]

The MCS feature of the model is not only useful for individual bank model applications but also for banking system models as the one presented in this paper. The MCS structure implies that weights are allowed to be *equation*-specific, not only variable-specific. A useful example to highlight that point is to consider the consolidated loan growth variable. With the MCS feature, economic activity variables on the right hand-side of the loan growth equations are weighed based on banks' (banking systems') exposure profiles, as they should, and not be based on trade (as in the traditional GVAR literature; as e.g. in Eickmeier and Ng (2011).[5] For banks it is not relevant how much the country in which they are located trades with another country. What matters for the consolidated bank when assessing its susceptibility to macro developments in other countries is instead its own exposure to the other country.

The paper is organized as follows. In Section 2 we present our dynamic model of optimal leveraging and explain how we quantify overleveraging. Section 3 is devoted to the threshold MCS-GVAR model, in which the overleveraging metric serves as the regime conditioning variable.

## 2 A theoretical model

Brunnermeier and Sannikov (2014) and Stein (2012) when studying overleveraging start with a continuous time model. We follow their approach but use a discrete time framework. In contrast to Mittnik and Semmler (2013) who also work with nonlinear finance-macro links, we, along the line of the previous literature, employ a stochastic model. For the solution of the model we use a novel method, called Nonlinear Model Predictive Control (NMPC), as presented in Gruene et al. (2015). In Annex A we present a brief introduction to how the NMPC methodology works. Commonly adopted dynamic policy models, such as DSGE models, tend to smooth out potentially destabilizing feedback mechanisms by assuming infinite-horizon decisions. Here, we propose a framework with a finite horizon that allows for vulnerabilities of financial intermediaries and destabilizing financial-macro feedback loops. It provides solutions that, when specifying a very long horizon, approach the usual infinite-horizon solutions. From this dynamic model empirical measures on overleveraging can be derived.

To introduce leveraging and net worth dynamics for financial institutions in a finite horizon decision model, we start with a low-dimensional stochastic model specification. The essential features of a model we employ can be found in Brunnermeier and Sannikov (2014) (Sec. 2), and also in Stein (2012).[6] Both specifications are stochastic, but they do not explicitly model macroeconomic feedback loops.

In this type of model, payout and leveraging are decision variables and the main state variable is net worth, namely $x_{1,t}$, in (3). In order to solve such a stochastic model through NMPC, one needs to add a stochastic shock sequence, see eq. (4), representing another state variable. In Brunnermeier and Sannikov (2014) capital returns are—due to capital gains—stochastic, as is the interest rate. BS start

[4]It is not the first application that combines a GVAR with a regime-switching mechanism; Binder and Gross (2013) combined a Markov-switching, i.e. continuous, regime switching structure with the GVAR.

[5]A likelihood ratio test for the predictive performance of all equations for the banking system variables in our model—once using the country-country weights and once the banking system - country weights—suggest that indeed the MCS structure with banking system - country weights is superior to the traditional GVAR with variable-specific weights for the very majority of banking systems. The test results are available upon request from the authors.

[6]For further details of the model and the derivation of the measures, see Schleer et al. (2014).

with a model where only the capital return is stochastic and they add a stochastic interest rate later by referring to time varying borrowing cost, reflecting the cost of screening and monitoring. Stein (2012) adds an important element to this type of model by providing a static solution for the model in terms of a mean variance approach that allows to measure excess leveraging.

While Brunnermeier and Sannikov (2014) specify their model in continuous time we here we adopt a discrete-time framework with a discounted instantaneous payout, $c_t$, and leverage, $\alpha_t$, as decision variables.[7] The model is stated in eqs. (1)–(4), where preferences are given by (1), the dynamics of the aggregate capital stock, $k_t$, by (2),[8] net worth, $x_{1,t}$, by (3), and the stochastic shock process, $x_{2,t}$, by (4). Adopting a discrete–time framework and decision horizon of $N$ periods, our model is given by

$$V = \max_{c_t, \alpha_t} E_t \sum_{t=0}^{N} \beta^t U(c_t x_{1,t}) \tag{1}$$

s.t.

$$k_{t+1} = k_t + h(g_t - \delta)k_t \tag{2}$$

$$x_{1,t+1} = x_{1,t} + hx_{1,t}[\alpha_t(y + \nu_1 ln x_{2,t} + r) + (1 - \alpha_t)(i - \nu_2 ln x_{2,t}) - \varphi(x_{1,t}) - c_t] \tag{3}$$

$$x_{2,t+1} \quad = \quad \exp(\rho \ln x_{2,t} + z_k). \tag{4}$$

In the above model (1)-(4), $c$ and $\alpha$ are our two decision variables, with the payout $c = C/x_1$, and $\alpha = 1 + f$, with $f = d/x_1$ the leverage ratio, measured as liability over net worth, $d$, is debt, $y$, the capital gains, driven by a stochastic shock, $\nu_1 ln x_{2,t}$.[9] Furthermore, $r$, is the return on capital, $i$, the interest rate paid on debt, also driven by a stochastic shock, $\nu_2 lng x_{2,t}$ [10], $\varphi(x_{1,t})$ is a convex adjustment cost, $h$, the time step size, $\rho$, a persistence parameter, with $\rho = 0.9$, and $z_k$ is an i.i.d. random variable with zero mean and a variance, $\sigma = 0.05$.

We can solve the model variant (1), (3) and (4) through a stochastic version of NMPC, see Gruene et al. (2015). Fig. 1 presents the path of the payout, $c_t$, light line, and leveraging $\alpha_t = 1 + f_t$, dark line. As is observable the stochastic capital gains and interest rates generate volatility of both payout and leveraging. We want to note that we solve here only for optimal leveraging. The payout tends to move with leveraging. Since $\alpha_t$ is a choice variable, both Brunnermeier and Sannikov (2014) and Stein (2012) assume that debt is redeemed in each period and, without cost, can be obtained on the market without frictions.

We also want to note that in BS there is only implicitly a macro feedback loop stylized, namely an externality, i.e., endogenous volatility, that is triggered below the steady state which makes the steady state unstable downward and not mean reverting as in Bernanke et al. (1999). In Brunnermeier and Sannikov (2014), feedback loops arise from large shocks below some steady state, triggering fire sales of assets, fall of asset prices and fall in net worth, generating a downward spiral.[11] Although model

[7] Note that BS (2014) use $x_t$ as a notation for their decision variable leveraging and assume that $x_t > 1$.

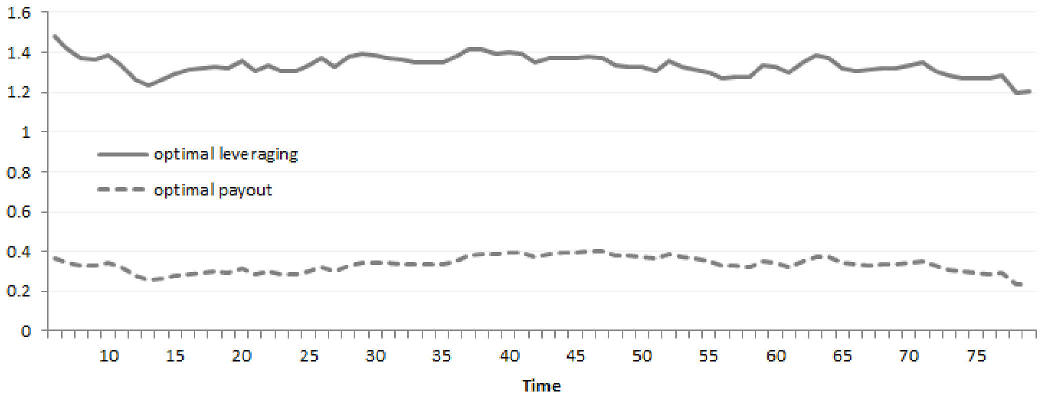
[8] In the solution procedure used here, we neglect (2). In BS, it represents the aggregate capital (with $g$ the growth rate and $\delta$ the resource use for managing the assets) of financial specialists and households. A larger fraction of it will be held by financial specialists, since they can borrow. Those details can be neglected here. Aggregate capital is more specifically considered in BS.

[9] Note that in our model version here, we neglect the dynamics of (2). In BS (2014), it represents the aggregate capital of financial specialists and households (with $g$ the growth rate of capital, another decision variable, and $\delta$ the resource use for managing the assets). A larger fraction of the assets will be held by financial specialists, since they have a lower discount rate, and with $\alpha_t > 0$, they can borrow. Those details can be neglected here. The dynamics of aggregate capital is more specifically considered in Brunnermeier and Sannikov (2014).

[10] Stein (2012) posits that the interest rate shocks are highly negatively correlated with capital gains' shocks, we have thus a negative sign in (3) for the effect of the shocks on interest rates. We here also assume that the interest rate shocks have smaller variance than the capital gains' shocks.

[11] In Stein the vulnerabilities and possibly adverse feedback loops are triggered by overleveraging, capital losses and rising borrowing cost.

Figure 1: Path of optimal payout, $c$, lower line, and optimal leveraging, $\alpha = (1 + f)$, upper line




(1)–(4) does not yet directly model instability, Fig. 1 depicts the volatility of $\alpha_t$, and thus the optimal leveraging $f_t$, and the payouts, $c_t$.[12]

Through our numerical computations we can also indirectly observe leveraging as defined in Brunnermeier and Sannikov (2014) as the ratio of assets to net worth: this is determined by the upper line in Fig. 1. As BS properly state, through leveraging, the capital share of banks in total capital – the share of financial experts in their terms – rises with leveraging, even at the stochastic steady state. This is also what creates in BS the source for endogenous risk.

Next we want to allow for regime change for the banks' fundamental variables such as the interest rate paid on bank liabilities. If there is a jump in the interest rate the bank has to pay for its debt, thus a jump in $i - \nu_2 lnx_{2,t}$, where the second expression would be negative, the optimal leveraging is likely to move down. Re-running now the stochastic model (1), (3) and (4) with low interest rates of $i = 0.02$, first and then switching to a high interest rate regime of $i = 0.12$ after 73 periods , in both cases with the stochastic shock added, we obtain the results as shown in Fig. 2. There we can observe how a jump of the interest rate – as the interbank borrowing and lending rate, for example represented by the TED spread– may impact the optimal leveraging.

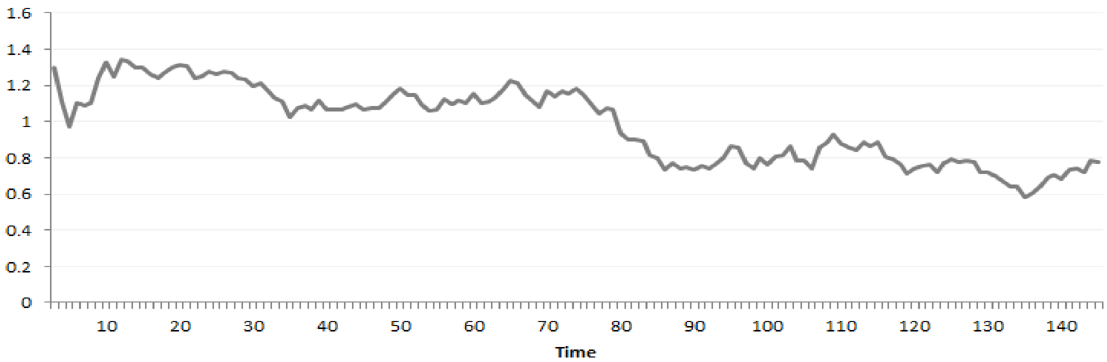
As can be observed from Fig. 2, there is a significant shift downward of the optimal leveraging, due to the interest rate jump, measured by our leverage ratio $1 + f_t$. A similar shift downward can be observed for net worth. As Fig. 2 shows, through the upward jump in the interest rate in period 73, the sustainable leveraging has clearly declined. Given some actual leveraging, the rise in the interest rate, fall in capital gains and fall in the sustainable (optimal) debt, the excess leveraging has now been increased.

These observable effects of a persistently higher interest rate might lead to the question of why the banks do not rationally expect this and adjust their actual leveraging appropriately. Since we work here with a model of finite time horizon, though there is some expectations on some shorter time horizon by banks, we implicity assume that there is some short-sightedness that allows the emergence of overleveraging and vulnerability of balance sheets of banks, that in fact makes our model more realistic compared to infinite time horizon models.[13]

[12]Excess leveraging in Stein (2012) is driven by actual leverage over and above the optimal leverage, caused by a sequence of persistent shock to increase capital gains and lowering interest rates, both giving rise to excess leveraging. In this sense, he explores only the vulnerability of the overleveraged sector, but does not model particular, possibly amplifying feedback loops, as Mittnik and Semmler (2013). However, Stein's model can distinguish between optimal debt, actual debt and excess debt. For empirical measures of those, see Schleer et al. (2014), and sect. 2.2.

[13]We want to thank a referee who has made this important point.

Figure 2: Regime change in interest rates and shift in optimal leverage $1+f_t$: upper line, up to period 73, leveraging for interest rate=0.02; lower line, from period 73, leveraging for interest rate 0.12




Similar results can also be derived from our static mean variance model of the next section. Thus when interest rates jump, this change is likely to add to the vulnerability of banks—since banks tend to become even more overleveraged because the sustainable leverage has declined.[14]

An empirical measure of overleveraging that we employ is based on the methodology developed by Schleer et al. (2014). In Annex B we sketch how the methodology works and refer to Schleer et al. (2014) for details that we omit. The methodology allows us to obtain estimates of optimal debt, and a deviation of actual to the estimated optimal level of debt along with it, to which we refer as our measure of overleveraging. We estimate the overleveraging indicator based on a sample of 40 EU banks and generate banking system aggregates (total asset-weighted aggregates).

# 3 The threshold MCS-GVAR model

## 3.1 Model structure

Our MCS-GVAR model comprises two cross-sections: $i = 1, ..., N$ EU countries and a cross-section of banking systems $j = 1, ..., M$. The endogenous variables belonging to the two cross-sections are collected in the vectors $\boldsymbol{x}_{it}$ and $\boldsymbol{y}_{jt}$, respectively. For a given cross-section item, the two vectors are of size $k_i^x \times 1$ and $k_j^y \times 1$.[15] The model has the following form:

$$\begin{aligned} \boldsymbol{x}_{it} &= \boldsymbol{a}_{ir} + \sum_{p_1=1}^{P_1} \boldsymbol{\Phi}_{i,p_1,r}\boldsymbol{x}_{i,t-p_1} + \sum_{p_2=0}^{P_2} \boldsymbol{\Lambda}_{i,0,p_2,r}\boldsymbol{x}^{*,C-C}_{i,t-p_2} + \sum_{p_3=0}^{P_3} \boldsymbol{\Lambda}_{i,1,p_3,r}\boldsymbol{y}^{*,C-B}_{i,t-p_3} + \boldsymbol{\epsilon}_{it} \\ \boldsymbol{y}_{jt} &= \boldsymbol{b}_{jr} + \sum_{q_1=1}^{Q_1} \boldsymbol{\Pi}_{j,q_1,r}\boldsymbol{y}_{j,t-q_1} + \sum_{q_2=0}^{Q_2} \boldsymbol{\Xi}_{j,0,q_2,r}\boldsymbol{x}^{*,B-C}_{j,t-q_2} + \sum_{q_3=0}^{Q_3} \boldsymbol{\Xi}_{j,1,q_3,r}\boldsymbol{y}^{*,B-B}_{j,t-q_3} + \boldsymbol{\omega}_{jt} \end{aligned} \tag{5}$$

The intercept terms $\boldsymbol{a}_{ir}$ and $\boldsymbol{b}_{jr}$ are of size $k_i^x \times 1$ and $k_j^y \times 1$, respectively. The two equation blocks contain a set of autoregressive terms — $(\boldsymbol{\Phi}_{i,1,r}, ..., \boldsymbol{\Phi}_{i,P_1,r})$ and $(\boldsymbol{\Pi}_{j,1,r}, ..., \boldsymbol{\Pi}_{j,Q_1,r})$ — which are

[14] For details of how those appear also in the overleveraging measure, see Schleer et al. (2014).
[15] For details that we omit, see Gross and Kok (2013) and Gross et al. (2016).

of size $k_i^x \times k_i^x$ and $k_j^y \times k_j^y$, respectively. The within- and across-cross-section dependence is introduced via the 'star' variable vectors. The corresponding coefficient matrices in the first equation block for the $\boldsymbol{x}_{it}$ — $(\boldsymbol{\Lambda}_{i,0,0,r}, ..., \boldsymbol{\Lambda}_{i,0,P_2,r})$ and $(\boldsymbol{\Lambda}_{i,1,0,r}, ..., \boldsymbol{\Lambda}_{i,1,P_3,r})$ — are of size $k_i^x \times k_i^{*x}$ and $k_i^x \times k_i^{*y}$. The corresponding coefficient matrices in the second equation block for the $\boldsymbol{y}_{jt}$ — $(\boldsymbol{\Xi}_{j,0,0,r}, ..., \boldsymbol{\Xi}_{j,0,Q_2,r})$ and $(\boldsymbol{\Xi}_{j,1,0,r}, ..., \boldsymbol{\Xi}_{j,1,Q_3,r})$ — are of size $k_j^y \times k_j^{*x}$ and $k_j^y \times k_j^{*y}$.

All coefficient matrices are assigned a subscript $r$ to indicate their regime dependence. Also the cross-section-specific shock vectors — $\boldsymbol{\epsilon}_{it}$ and $\boldsymbol{\omega}_{jt}$ — which are of size $k_i^x \times 1$ and $k_j^y \times 1$ — have regime-dependent covariance matrices $\boldsymbol{\Sigma}_{ii}^x$ and $\boldsymbol{\Sigma}_{jj}^y$, along with zero means. A global matrix $\boldsymbol{\Sigma}$ captures the covariance structure of the combined set of residuals from the two equation blocks under an assumed *regime constellation* (more on that below).

The regimes are defined as a function of the banking system-specific overleveraging indicator that was presented in the previous section. It is a discrete, for the time being exogenous, switching scheme, defining the two regimes for a given banking system and corresponding country as Regime 1 when overleveraging is negative (reflecting underleveraging), and as being in Regime 2 when the overleveraging indicator is larger than zero.

There are six variables involved in the model: three in the country cross-section and three in the bank cross-section. The country cross-section vector includes nominal GDP, a GDP deflator, and long-term interest rates. GDP and the GDP deflator are modeled in quarter-on-quarter (QoQ) differences of natural log levels. Long-term interest rates are modeled in QoQ differences.

The three endogenous variables for the banking systems include: Nominal credit, nominal loan interest rates and a capital ratio.[16] Nominal credit volumes are sourced from the ECB's Balance Sheet Statistics, which capture domestic lending as well as direct cross-border lending to households and non-financial corporations (i.e. the measure is not purely locational). The capital ratio is defined as equity capital over total assets. Nominal credit is modeled in QoQ differences of natural log levels. Loan interest rates and the capital ratio are modeled in QoQ differences.

The variable vectors that are assigned an asterisk in eq. (9) need to be generated by means of a set of weights that link the items within and across the cross-sections. We outline the weight settings briefly in the following and refer to Gross et al. (2016) for details.

**Countries — Countries ($\boldsymbol{W}^{C-C}$)**: A measure of *bilateral trade* (sum of nominal imports and exports between any two countries) is used to calibrate the cross-country weights. The weight of a country to itself is zero at any point in time. The trade data is sourced from the IMF trade statistics. It has an annual frequency which is interpolated to quarterly by means of a quadratic match sum conversion method.

**Banking systems — Countries ($\boldsymbol{W}^{B-C}$)**: The weights are calibrated based on BSI domestic and cross-border credit exposures.

**Countries — Banking systems ($\boldsymbol{W}^{C-B}$)**: Can be seen as the mirror of the weights for linking countries to banking systems ($\boldsymbol{W}^{B-C}$), i.e. $\boldsymbol{W}^{C-B}$ will be the transpose of the bank-country matrix for every quarter over the sample period.

**Banking systems — Banking systems ($\boldsymbol{W}^{B-B}$)**: We employ domestic and cross-border credit exposures to financial corporations as a basis for calibrating these weights.[17]

[16] We do not yet capture the foreign operations of banks via subsidiaries and branches (as we use the ECB's BSI statistics, i.e. a locational concept of monetary financial institutions). We have compared the exposure profile (weights) based on the BSI statistics with the weights using consolidated banking data and they are broadly aligned. However, to fully capture the cross-border lending activity, our model will need to be further extended and use the consolidated MFI groups, whereby individual MFIs are associated with their parents. This is work in progress.

[17] Despite the fact that our measure of loan volume in the core of the model captures only credit to the private sector we use a banking system exposure measure to generate the weights that link the banking systems. The rationale is

Two of the four weight matrices are square matrices that have zero entries on their diagonals at every point in time, namely the $\boldsymbol{W}^{C-C}$ and $\boldsymbol{W}^{B-B}$ matrix. The other matrices, cross-linking the cross-sections, are not square unless the number of items in two sections would be equal. Moreover, their diagonals do not need to equal zero. The model set-up is flexible in the sense that countries can be included in the model for which there are no corresponding banking systems. Vice versa, banking systems could be included in the model for which the corresponding country would not be included. An asymmetry is in fact present in our model application as 28 EU economies are covered in the country cross-section while having only 14 banking systems for which the overleveraging indicator is available.

The model is estimated based on data covering the 1995Q1-2014Q4 period (80 observations). All weight matrices are time-varying over the whole sample period. The model has $3 \times 28 + 3 \times 14 = 126$ equations which are all individually estimated by means of an Iteratively Reweighted Least Squares (IRLS) method. The method is more robust to outliers than Ordinary LS and helps stabilize the dynamics of the global model. Each equation is estimated twice, conditional on the banking system and corresponding country being either in Regime 1 or 2. The derivation of the global solution of the MCS-GVAR model is outlined in Annex C. Details that are omitted there can be found in Gross et al. (2016).

Instead of the piece-wise linear and for the time being exogenous regime switching structure that we have chosen to augment an otherwise standard MCS-GVAR model was deliberate. Other nonlinear model structures, such as Markov-switching, smooth transition, or unconditional time-varying parameter model structures could have been embedded in the GVAR. The choice of a *conditional* regime dependence was deliberate since we want to reflect the specific theory that we have put forward in the model, i.e. implying the dependence of the system dynamics specifically on the deviation of leverage from its optimal level.

## 3.2 Empirical results

Our empirical assessment based on the T-MCS-GVAR model relies on sign-restricted impulse responses. We introduce three different shock types, the first two of which involve sign constraints.[18] All three shock types—to which we refer as Type 1, Type 2, and Type 3—start from the same positive capital ratio shock of a banking system-specific amount $\Delta$.

**Contractionary deleveraging** (Type 1): Banks (banking systems) are assumed to move to the higher capital ratio by shrinking their balance sheet, with equity capital by assumption being constant and debt shrinking along with assets.

**Expansionary deleveraging** (Type 2): Banks (banking systems) are assumed to raise equity capital and invest the additional funds, while holding debt constant by assumption.[19]

**Unconstrained deleveraging** (Type 3): Banks (banking systems) are not constrained as to how they move to a higher capital ratio. They may partly shrink their balance sheet size or raise equity.[20]

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that a shock propagation channel for loan volumes and prices toward the private sector can nonetheless be a function of the size of interbank exposure. Various robustness checks with different weighting schemes in particular in this respect confirm that our simulation results are robust.

[18] As an entry point to the literature about sign restricted SVARs see Faust (1998), Canova and Nicolo (2002), and Uhlig (2005).

[19] We shall note that the Type 2 scenario does not literally reflect the creation of new loans up to the full potential that some new equity capital would imply. For that reason, the positive Type 2 impulse likely underestimates the upside potential of the additional credit supply as a result of higher capitalization levels. For that reason, less emphasis should be given to the Type 2 scenario, and more to the Type 1 and 3 scenario simulations.

[20] An additional, fourth simulation type could in principle be one under which banks are assumed to raise capital to replace debt, while holding total assets constant by assumption. This scenario is one that can be called a *static*

The Type 1 and 2 shock sizes are calibrated based on the formulas in eqs. 6 and 7, with $E_0$, $A_0$, and $\Delta$ denoting capital, total assets, and the capital ratio shock respectively:

$$shock^{Type1} = \ln\left(\frac{E_0}{\frac{E_0}{A_0}+\Delta}\right) - \ln(A_0) \tag{6}$$

$$shock^{Type2} = \ln\left(A_0 - E_0\left(\frac{\left(\Delta+\frac{E_0}{A_0}\right)(A_0-E_0)}{E_0\left(\Delta+\frac{E_0}{A_0}-1\right)}+1\right)\right) - \ln(A_0) \tag{7}$$

The respective first terms in the two equations reflect the total asset values after the capital ratio shock $\Delta$ is applied. The shock amounts equal the log difference between total assets post- and pre-shock. The log percentage shock computed based on capital and total assets is assumed to apply to the loan stock of a country.

The capital ratio shock under the Type 1 simulation is combined with the sign restriction imposed on banking system loan volumes and loan interest rates, which are assumed to be negative and positive, respectively, for only the first period in which the shock arrives. This combination of sign constraints makes the impulse an identified negative loan supply shock.

Under the Type 2 simulation, the same capital ratio shock is combined with the opposite of the Type 1 restrictions, i.e. a positive and negative constraint on the $T = 1$ responses for loan volumes and rate. It is therefore an identified positive supply shock.[21] [22]

The Type 3 simulation—without the imposition of any sign constraints —is meant to reveal how banking systems went about the deleveraging process on average historically. It is a Generalized Impulse Response (G-IR) simulation scheme that we employ for Type 3 (Koop et al. (1996)). Note, moreover, that for both the Type 1 and 2 simulation schemes there are no constraints imposed on the responses of nominal GDP, the GDP deflator, and long-term interest rates.

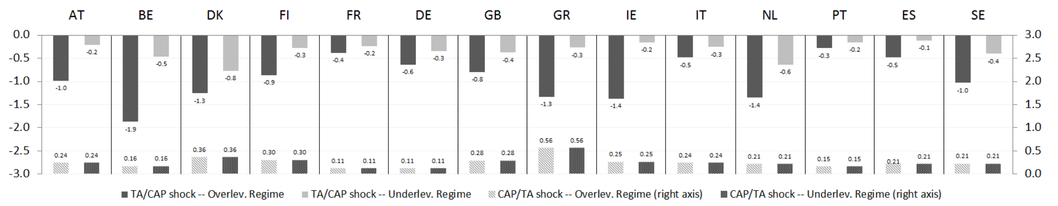
Along with the three different simulation types, we distinguish between two regime constellations. A first is to set all countries and banking systems simultaneously to Regime 1, i.e. for all to jointly rest in the underleveraging regime. The second constellation is to assume them all simultaneously being in Regime 2, the overleveraging regime. Many more regime constellations would be conceivable, with subsets of countries and banking systems being either in the over- or under-leveraging regime. We do not present them here and focus instead on the two polar constellations. Prior to the outbreak of the financial crisis period (2007) all countries comprised by our sample rested in the overleveraging

*deleveraging*, which is not very relevant, as too hypothetical and not implying structural changes, for what concerns the empirical assessment that we wish to conduct.

[21] The identification scheme ignores the fact that while banks having raised equity should be in a position to generate more credit, hence pushing lending rates down, the additional equity will also imply a dilution of existing shareholders and reduce the Return on Equity (RoE). This could be expected to induce banks to increase lending margins in order to reinstall their desired ROE target. By only imposing the sign restriction in the first period, our simulations do want to allow for such rent-seeking behavior in subsequent periods. We shall, moreover, note that the Type 2 behavior of banks would not only reflect equity raising activities but as well a gradual rise in capital levels by retaining earnings. Moreover, we see one useful extension of the model that would allow refining the identification of bank deleveraging, once we augment the model by some non-bank credit volumes and/or prices that can serve as substitutes for bank credit (e.g. corporate bonds). One can add to the identification of bank credit supply shocks by constraining non-bank aggregates to substitute bank supply, i.e. in the case of a negative (Type 1) bank supply shock e.g. by constraining non-bank credit volumes to expand and their prices to fall. We leave the inclusion of a non-bank (shadow-bank) sector for a future extension of the model.

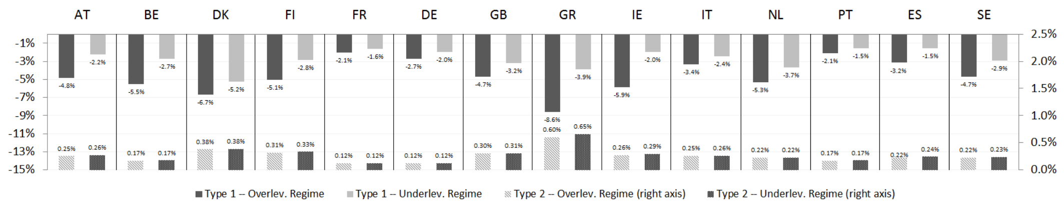
[22] While we do aim to emphasize with the Type 2 reaction of banks that raising equity to achieve higher capital ratios can be useful to avoid an asset side deleveraging and implied contraction of economic activity, we abstract from whether and to what extent this would be possible, in particular during recession periods in general, or banking crises episodes specifically, when raising capital from the market from all banks simultaneously might prove difficult.

Figure 3: Capital ratio and implied leverage shocks under over- and under-leveraging




**Note:** Residual-based 1-standard deviation shocks for capital ratios.

Figure 4: Capital ratio shock-implied loan supply shocks under Type 1 and 2 simulation




regime (except Finland), thus the all-in-overleveraging regime comes close in fact to an observed regime constellation.

The shock sizes are calibrated based on the full-sample residuals (1-standard deviation based), i.e. are on purpose not made regime-conditional in order to thereby be able to judge the difference of responses due to Type 1/2/3 and due to the different regime dynamics. The starting point capital ratios to which the shocks are attached are regime conditional, however, by first identifying the two points in time when the capital ratio of a banking system came closest to the average capital ratio under the overleveraging and underleveraging regime, respectively, for each banking system.

Fig. 3 shows the capital and implied leverage ratio (assets over equity capital) shocks under the two regimes for all banking systems. The capital ratio shocks are the same under the two regimes. The implied shocks to the leverage multiples are, however, different which reflects the effect of levered balance sheets. During low leverage (high capital ratio) periods, the same percentage point shock to capital ratios has less of an effect on leverage.

Fig. 4 shows the capital ratio shock-implied shock sizes for loan volumes under the Type 1 and 2 simulation schemes. The absolute shock sizes under the Type 1 simulation are larger than the corresponding absolute shock sizes under Type 2. There is a pronounced difference between implied loan volume shocks for Type 1 when differentiating between the initial overleveraging versus under-leveraging regime. Under the overleveraging (too low capital ratio) regime, the implied shock sizes to credit are more sizable than under initial underleveraging.

We present the model responses for real GDP of five example simulations in Fig. 5—applying positive capital ratio shocks to the banking systems of Austria, Belgium, France, Ireland, and the Netherlands. The time dimension (12 quarters) is compressed by showing only the cumulative long run effects. Along with the domestic responses, the cross-border responses are compressed by computing weighted aggregate responses after having obtained the full cross-country response profiles from the simulation, using the cross-border loan exposures for defining the weights.[23] The responses of real

[23]GDP-based, i.e. size-of-the-economy-based weights would be an alternative. It is a matter of preference. The exposure-based weights mean that the results reflect the response of countries that are relevant for the banking system

GDP are obtained by subtracting the responses of the GDP deflator from those of nominal GDP responses. The error bounds around the cumulative responses in Fig. 5 mark the 80th and 20th percentiles of the response distributions, reflecting parameter uncertainty as well as the draws from the sign restriction procedure.

For Austria, the initial capital ratio shock (+0.24pp) which implied a loan supply shock under the Type 1 simulation equalling -4.8pp and -2.2pp under the over- and under-leveraging regime, would result in a GDP drop relative to cumulative long-run baseline growth of about -2.5pp and -0.5pp under the two regimes. Let us recall that these estimates are conditional on the assumption that *all* banking systems are simultaneously in the over- or underleveraging regime, respectively, not only the Austrian one. Looking at the Type 1 domestic effects, the clear asymmetry that was visible for the underlying loan supply shocks feeds, as expected, through to the GDP responses.

Under the Type 2 simulation the effects are slightly positive, reflecting the underlying positive supply shock. The responses of GDP or the deflator were not constrained, i.e. the fact that real GDP responses are negative and positive respectively under Type 1 and 2 is driven by the stable relation between credit and GDP. The Type 2 effects for real GDP are less sizable in absolute terms compared to the Type 1 effects, being reflective once again of the asymmetric supply shock sizes. The Type 3 responses fall between the Type 1 and 2 estimates, equalling -0.6pp and -0.3pp under the two regime constellations. For the case of the Austrian banking system shock, the weighted cross-border real GDP effects appear to be relatively limited, with the maximum effect at -0.3pp under the overleveraging regime for all banking systems coupled with the assumed Type 1, i.e. full asset-side deleveraging, reaction of the Austrian banking system. The remaining cases, with shocks applied to Belgium, France, Ireland, and the Netherlands, suggest the same response patterns conditional on the over- versus underleveraging regime constellation.

Fig. 6 shows the real GDP to regime-conditional nominal credit long-run impact ratios for the five example shock scenarios. They are the same for the Type 1 and 2 simulation (as the two are operating with identical shocks and just opposite sign restrictions).

The GDP to credit impact ratios suggest that with the exception of only France, the ratio is higher under the over- than under the under-leveraging regime. This suggests that there is a stronger relation between credit and GDP under the overleveraging regime, which is an effect that is independent of, i.e. to be seen in addition to, the aforementioned mechanic effect of levered balance sheets that imply different supply shock sizes for the same capital ratio shock.

that is shocked in terms of their exposure. GDP-based weights would look at the results from the viewpoint of size-relevance.

Figure 5: Shock to banking system capital ratios — Real GDP responses

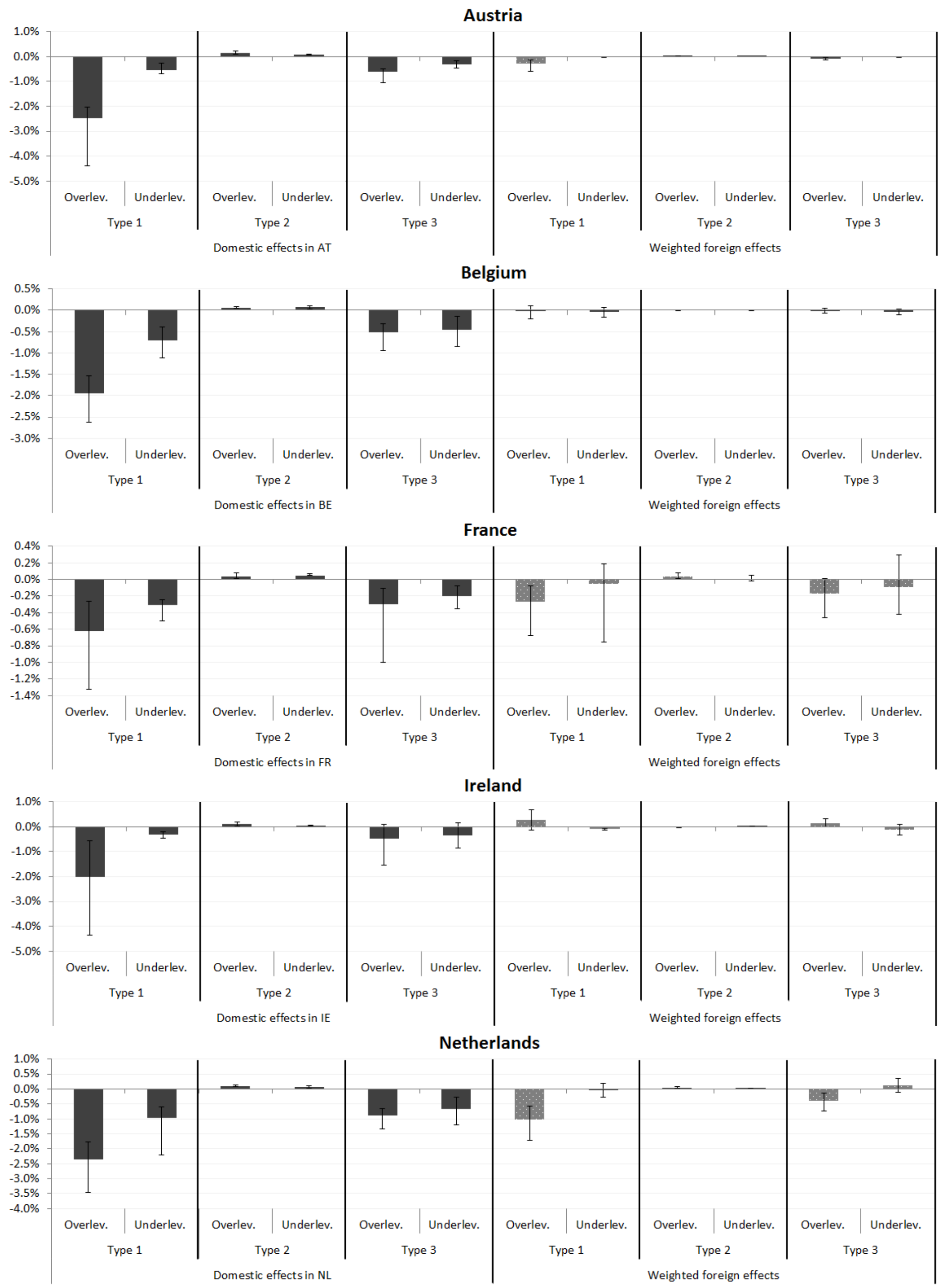
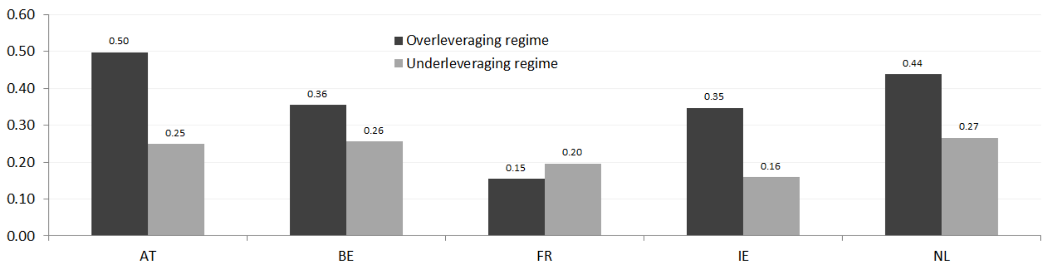


Figure 6: Real GDP to nominal credit long run impact ratios




# 4 Conclusions

Much recent research point to the excessive leveraging of the banking sector as being the essential driving force for macroeconomic instability. We show here, based on a model by Brunnermeier and Sannikov (2014) and Stein (2012), how to measure overleveraging and build it into a large-scale macro model. We pursue this from an empirical perspective for a sample of European countries where we aggregate forty banks' individual measures of overleveraging for 14 banking systems and use the overleveraging measure as a transition variable in a Threshold Mixed-Cross-Section GVAR for 28 countries and 14 banking systems of the EU.

A first conclusion is that one reason for why macroeconomic responses to bank capital shocks are likely significantly stronger under an overleveraging regime is that the same percentage point capital ratio shock translates into a much stronger asset side reaction (which is particularly visible under the Type 1 simulation) than under an initial low-leverage (i.e. high capital ratio) regime. This effect is very much mechanical, though quite essential.

Second, our observation is that GDP to credit long-run response ratios are higher under a bank overleveraging regime. It is an effect that arises in addition to the shock amplification effect (previous point), whereby credit and GDP tend to co-move more strongly, hence rendering economic activity more susceptible to changes in bank credit supply under the overleveraging regime.

Third, cross-border spillover effects appear to be more pronounced when capital ratio shocks hit the banking system during a period of overleveraging. This finding reflects the fact that bank credit supply does not only exert its state-dependent effect on the domestic economy but implies cross-border effects through the cross-border supply of credit.

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# Annex A: The numerical procedure

For the numerical solution of our dynamic decision problem we employ a new procedure. Usually, DYNARE or Dynamic Programming (DP) are used to solve models of the type that we have presented in Section 2. DYNARE frequently employs first or higher-order approximations to the steady state to solve the model. DP can provide global solutions, but the disadvantage of DP is that its computational burden tends to grow exponentially with the dimension of the state variable. Hence, even for moderate state dimensions, it may be virtually impossible to obtain a reasonably accurate solution.

For this reason, we employ a method called Nonlinear Model Predictive Control (NMPC) that was developed in Gruene et al. (2015). Instead of computing the optimal value function for all possible initial states, NMPC only computes single trajectories. To describe the method, consider the generic dynamic decision problem

$$\text{maximize} \quad \int_0^N e^{-\rho t} \ell(x(t), u(t)) dt,$$

where $x(t)$ satisfies $\dot{x}(t) = f(x(t), u(t))$, $x(0) = x_0$ and the maximization takes place over a set of admissible decision functions. By discretizing this problem in time, we obtain an approximate discrete time problem of the form

$$\text{maximize} \quad \sum_{i=0}^{N} \beta^i \ell(x_i, u_i) dt, \tag{8}$$

where the maximization is now performed over a sequence $u_i$ of decision values and the sequence $x_i$ satisfies $x_{i+1} = \Phi(h, x_i, u_i)$, Here $h > 0$ is the discretization time step, $\beta = e^{-\rho h}$ and $\Phi$ is a numerical scheme approximating the solution of $\dot{x}(t) = f(x(t), u(t))$ on the time interval $[ih, (i+1)h]$.

The idea of NMPC is to replace the maximization of the above large horizon functional, where we could have $T \Rightarrow \infty$, by the iterative maximization of finite horizon functionals

$$\text{maximize} \quad \sum_{k=0}^{N} \beta^i \ell(x_{k,i}, u_{k,i}) dt, \tag{9}$$

for a truncated finite horizon $N \in \mathbb{N}$ with $x_{k+1,i} = \Phi(h, x_{k,i}, u_{k,i})$ and the index $i$ indicates the number of the iteration, cf. the algorithm below. Note that neither $\beta$ nor $\ell$ nor $\Phi$ changes when passing from eq. (8) to (9), only the horizon is truncated.

Problems of the type in eq. (9) can be efficiently solved numerically by converting them into a static nonlinear program and solving them by efficient NLP solvers, see Gruene and Pannek (2011). In our simulations, we have used a discounted variant of the MATLAB routine which uses MATLAB's `fmincon` NLP solver in order to solve the resulting static optimization problem.

Given an initial value $x_0$, an approximate solution of (8) can now be obtained by iteratively solving (9) as follows:

(1) for i=1,2,...
(2) solve (9) with $x_{0,i} := x_i$, denote the resulting optimal control sequence by $u^*_{k,i}$
(3) set $u_i := u^*_{0,i}$ and $x_{i+1} := \Phi(h, x_i, u_i)$
(4) end of for-loop

This algorithm yields an infinite trajectory $x_i$, $i = 1, 2, \ldots$ whose control sequence $u_i$ consists of all the first elements, $u^*_{0,i}$, of the decision sequences for the finite horizon subproblems (9).

Under suitable assumptions, it can be shown that the solution $(x_i, u_i)$ (which depends on the choice of $N$ above) converges to the correct solution of (8) as $N \to \infty$. The main assumptions is the existence of an equilibrium for the infinite horizon problem (8). If the equilibrium is known, it can be used as an additional constraint in (9), in order to improve the convergence properties. For our optimization, we make use of recent results showing that convergence can be ensured even without a priori knowledge of the equilibrium, see Gruene et al. (2015).

## Annex B: A measure of overleveraging

One can show how the optimal debt ratio can directly be derived in the simplified case of logarithmic utility using the model (1), (3) and (4).[24] The state variable is a stochastic differential equation in net worth as defined in eq. (3).

$$dX(t) = X(t)[(1 + f(t))(dP(t)/P(t) + \beta(t)dt) - i(t)f(t) - cdt], \tag{10}$$

where $X(t)$ is net worth[25], $f(t) \equiv L(t)/X(t)$ debt over net worth; thus leverage, $dP(t)/P(t)$ denotes stochastic capital gains or losses, and $i(t)$ the interest rate, also stochastic. Assets over net worth are defined as $(1 + f(t))$, $\beta(t)$ is the trend in bank capital returns, and $C(t)/X(t) \equiv c(t)$ is consumption over net worth. In the derivation, $c$ is taken as given.

The optimal debt ratio $f^*$ maximizes the difference between the mean $M(f(t))$ and risk $R(f(t))$ which represents a mean-variance formulation. Thus, we obtain:

$$f^* = argmax[M(f(t)) - R(f(t))] = [(a(t) + \beta(t) - i) - (\sigma_p^2 - \rho\sigma_i\sigma_b)]/\sigma^2 \tag{11}$$

where $\sigma$ is a risk element given by $\sigma^2 = \sigma_i^2 + \sigma_r^2 - 2\rho\sigma_i\sigma_r$, and $\sigma_i^2$ is the variance of the interest rate, $\sigma_r^2$ is the variance of capital gains, and $\rho$ defines the correlation of $i$ and $r$. In this model variant, Stein (2012) assumes that the asset price grows at a trend rate $r$ and there is a deviation from the trend, $y$:

$$dP(t)/P(t) = (r + \alpha(0 - y))dt + \sigma_r dw_p \tag{12}$$

Moreover, $\beta(t)$, the return on capital, is considered as deterministic. The optimal debt or leverage ratio $f^*(t)$ is then defined as follows:

$$f^*(t) = [(r - i) + \beta - \alpha y(t) - \frac{1}{2}\sigma_r^2 + \rho\sigma_i\sigma_r]/\sigma^2, \tag{13}$$

where $i$ is the credit cost of banks, $\beta$ the trend return on capital, and $y(t)$ is the deviation of capital gains from its trend. Then follows the variance term with the parameters as indicated above.

Given those measures, one can employ data to derive the optimal debt ratio for a sample of EU banks. We rely on data obtained from Thomson-Reuters-Datastream. The optimal debt ratio is normalized by calculating the difference between the optimal debt ratio and the mean over the period and dividing it by the standard deviation. The actual debt ratio is calculated as long-term debt over total assets. Again, the actual debt ratio is also normalized to match its unit to the one of the optimal debt ratio and then the optimal debt is subtracted from it to obtain the overleveraging.

[24] For a complete description of the derivation relying on Model I of Stein (2012) (ch. 4.9), and Schleer et al. (2014).

[25] Since we have here only one state variable, we are using $X(t)$ now as net worth.

# Annex C: Global solution of the MCS-GVAR model

The system presented in the set of eq. 5 contains time-contemporaneous relationships and hence is not yet ready for being used for forecasting or simulation purposes. The global model has therefore to be solved, meaning that the equations for all countries and banking systems need to be stacked and then reformatted in a way to contain only lagged relationships. The global solution of the model can be derived in four steps.

**Step 1: Generate A-matrices.** We start by stacking the within-cross-section vectors along with the cross-cross-section weighted variable vectors in (here) two vectors $\boldsymbol{m}^x_{it}$ and $\boldsymbol{m}^y_{jt}$. In this first step, the fact that the model has a threshold structure does not have any implication yet.

$$\begin{aligned} \boldsymbol{m}^x_{it} &= \begin{pmatrix} \boldsymbol{x}_{it\prime} & \boldsymbol{x}^{*,C-C\prime}_{it} & \boldsymbol{y}^{*,C-B\prime}_{it} \end{pmatrix}' \\ \boldsymbol{m}^y_{jt} &= \begin{pmatrix} \boldsymbol{y}_{jt\prime} & \boldsymbol{x}^{*,B-C\prime}_{jt} & \boldsymbol{y}^{*,B-B\prime}_{jt} \end{pmatrix}' \end{aligned} \tag{14}$$

We can rewrite the equation system with these $\boldsymbol{m}$ vectors as follows.

$$\begin{aligned} \underbrace{\begin{pmatrix} I_{k^x_i} & -\Lambda_{i,0,0} & -\Lambda_{i,1,0} \end{pmatrix}}_{\equiv \boldsymbol{A}^x_{i0}} \boldsymbol{m}^x_{it} &= \boldsymbol{a}_i + \underbrace{\begin{pmatrix} \Phi_{i1} & \Lambda_{i,1,1} \end{pmatrix}}_{\equiv \boldsymbol{A}^x_{i1}} \boldsymbol{m}^x_{i,t-1} + ... + \boldsymbol{\epsilon}_{it} \\ \underbrace{\begin{pmatrix} I_{g^y_j} & -\Xi_{j,0,0} & -\Xi_{j,1,0} \end{pmatrix}}_{\equiv \boldsymbol{A}^y_{j0}} \boldsymbol{m}^y_{jt} &= \boldsymbol{b}_j + \underbrace{\begin{pmatrix} \Pi_{j1} & \Xi_{j,1,1} \end{pmatrix}}_{\equiv \boldsymbol{A}^y_{j1}} \boldsymbol{m}^y_{j,t-1} + ... + \boldsymbol{\omega}_{jt} \end{aligned} \tag{15}$$

At this point, in eq. 15, a choice has to be made for the regime-dependent coefficient matrices. In order to not overload the notation, we do not introduce any additional scripting but note that a regime choice has to be made here. For each country or banking system, either Regime 1 or Regime 2 conditional parameters shall be set. Overall, for across countries and banking systems it is therefore a *regime constellation* that should be defined. Each regime constellation implies different global model dynamics.

**Step 2: Generate L-matrices (link matrices).** With a global, stacked variable vector $\boldsymbol{s}_t = (\boldsymbol{x}'_{1t}, ..., \boldsymbol{x}'_{Nt}, \boldsymbol{y}'_{1t}, ..., \boldsymbol{y}'_{Mt})$ at hand, we can link the cross-section-specific variable vectors $\boldsymbol{m}^x_{it}$ and $\boldsymbol{m}^y_{jt}$ to $\boldsymbol{s}_t$. The link matrices $\boldsymbol{L}^x_i$ and $\boldsymbol{L}^y_j$ are used to map the local cross-section variables into the global vector, which involve the weights from the weight matrices $\boldsymbol{W}$.

$$\begin{aligned} \boldsymbol{m}^x_{it} = \boldsymbol{L}^x_i \boldsymbol{s}_t \quad &\rightarrow \quad \boldsymbol{A}^x_{i0}\boldsymbol{L}^x_i \boldsymbol{s}_t = \boldsymbol{a}_i + \boldsymbol{A}^x_{i1}\boldsymbol{L}^x_i \boldsymbol{s}_{t-1} + ... + \boldsymbol{\epsilon}_{it} \\ \boldsymbol{m}^y_{jt} = \boldsymbol{L}^y_j \boldsymbol{s}_t \quad &\rightarrow \quad \boldsymbol{A}^y_{j0}\boldsymbol{L}^y_j \boldsymbol{s}_t = \boldsymbol{b}_j + \boldsymbol{A}^y_{j1}\boldsymbol{L}^y_j \boldsymbol{s}_{t-1} + ... + \boldsymbol{\omega}_{jt} \end{aligned} \tag{16}$$

**Step 3: Generate G-matrices.** The equation-by-equation system can now be stacked into a global system.

$$
\boldsymbol{G}_0^x = \begin{pmatrix} \boldsymbol{A}_{10}^x \boldsymbol{L}_1^x \\ ... \\ \boldsymbol{A}_{N0}^x \boldsymbol{L}_N^x \end{pmatrix}, \boldsymbol{G}_1^x = \begin{pmatrix} \boldsymbol{A}_{11}^x \boldsymbol{L}_1^x \\ ... \\ \boldsymbol{A}_{N1}^x \boldsymbol{L}_N^x \end{pmatrix}, ..., \boldsymbol{a} = \begin{pmatrix} \boldsymbol{a}_1 \\ ... \\ \boldsymbol{a}_N \end{pmatrix}
$$

$$
\boldsymbol{G}_0^y = \begin{pmatrix} \boldsymbol{A}_{10}^y \boldsymbol{L}_1^y \\ ... \\ \boldsymbol{A}_{M0}^y \boldsymbol{L}_M^y \end{pmatrix}, \boldsymbol{G}_1^y = \begin{pmatrix} \boldsymbol{A}_{11}^y \boldsymbol{L}_1^y \\ ... \\ \boldsymbol{A}_{M1}^y \boldsymbol{L}_M^y \end{pmatrix}, ..., \boldsymbol{b} = \begin{pmatrix} \boldsymbol{b}_1 \\ ... \\ \boldsymbol{b}_M \end{pmatrix} \tag{17}
$$

These cross-section-specific $G$ matrices can be further combined to a set of global $G$ matrices. The intercept vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are combined in a vector $\boldsymbol{d}$. That is,

$$
\boldsymbol{G}_0 = \begin{pmatrix} \boldsymbol{G}_0^x \\ \boldsymbol{G}_0^y \end{pmatrix}, \boldsymbol{G}_1 = \begin{pmatrix} \boldsymbol{G}_1^x \\ \boldsymbol{G}_1^y \end{pmatrix}, ..., \boldsymbol{d} = \begin{pmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{pmatrix} \tag{18}
$$

**Step 4: Generate H-matrices.** The global system can now be pre-multiplied by the inverse of $\boldsymbol{G}_0$. The system is now ready to be used for shock simulation and forecast purposes.

$$
\boldsymbol{s}_t = \underbrace{G_0^{-1} d}_{\equiv H_0} + \underbrace{G_0^{-1} G_1}_{\equiv H_1} \mathbf{s}_{t-1} + ... + \boldsymbol{G}_0^{-1} \boldsymbol{\varphi}_t \tag{19}
$$

It is useful to recall at this point that the coefficient matrices $H$ are the result of the above-mentioned assumption about a *regime constellation* across countries and banking systems. Moreover, since the weights are time-varying a choice has to be made as to the reference point in time of which the weights are taken to solve the regime-constellation dependent global model. The shock simulations that are presented in this paper take the end-sample weight sets as a basis for deriving the global solution.

**Marco Gross**
European Central Bank, Frankfurt am Main, Germany; email: marco.gross@ecb.europa.eu

**Jérôme Henry**
European Central Bank, Frankfurt am Main, Germany; email: jerome.henry@ecb.europa.eu

**Willi Semmler**
University of Bielefeld, Bielefeld, Germany, The New School, New York, United States; email: semmlerw@newschool.edu



Postal address 60640 Frankfurt am Main, Germany
Telephone +49 69 1344 0
Website www.ecb.europa.eu



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