# What Can Measured Beliefs Tell Us About Monetary Non-Neutrality?

Hassan Afrouzi

Joel P. Flynn

#### **Choongryul Yang**

Columbia University & NBER Yale University

Federal Reserve Board

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- On the information frictions side, survey data has produced ample micro evidence that the average firm is highly uncertain about economic outcomes
- Missing link: understanding how firms' measured beliefs can be used to assess the importance of information frictions for monetary non-neutrality
- This Paper: How do individual firms' beliefs map to monetary non-neutrality?
  - Whose expectations matter for real effects of monetary shocks and how?
  - How can we *measure* the role of information frictions (using survey data)?

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- Theorem 2: Sufficient statistic for monetary non-neutrality only depends on subjective uncertainty of most informed firms
- Theorem 3: Data on beliefs are not only sufficient but also necessary
- Quantitative: Measure monetary non-neutrality using survey data on uncertainty
  - Informational frictions approximately double monetary non-neutrality
  - But selection dampens it by approximately 50%

### Literature

- Models of rational inattention
  - Sims (2003, 2010); Maćkowiak and Wiederholt (2009, 2015); Afrouzi and Yang (2021)
    - $\rightarrow$  No nominal rigidities
- Models of observation costs + menu costs and monetary non-neutrality
  - Reis (2006); Alvarez, Lippi, and Paciello (2011, 2016)
    - $\rightarrow$  Perfect info conditional on observation.
- Informational Foundations of Nominal Rigidities
  - Woodford (2009); Stevens (2020); Morales-Jiménez and Stevens (2024)
    - $\rightarrow$  Key difference in our model: agents' prior beliefs evolve over time
- Using data on beliefs to quantify monetary non-neutrality
  - Roth, Wiederholt, and Wohlfart (2023); Afrouzi (2024); Yang (2022)
- Survey evidence on firms' beliefs
  - Coibion, Gorodnichenko, Kumar, and Ryngaert (2021) among many others

# Model

#### Household

$$\max \int_{0}^{\infty} e^{-rt} \left( \frac{C_{t}^{1-\gamma} - 1}{1-\gamma} + \log(\frac{M_{t}}{P_{t}}) - \alpha L_{t} \right) dt$$
  
s.t.  $C_{t} = \left( \int_{0}^{1} A_{i,t}^{\frac{1}{\eta}} C_{i,t}^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}$   
 $M_{0} + \int_{0}^{\infty} e^{-\int_{0}^{t} R_{s} ds} \left( w_{t} L_{t} - \int_{0}^{1} P_{i,t} C_{i,t} di - R_{t} M_{t} + \Pi_{t} \right) dt = 0$ 

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• Monetary Policy:  $M_t = \overline{M} \implies R_t = r$ 

Household

$$\begin{split} &\max \int_{0}^{\infty} e^{-rt} \left( \frac{C_{t}^{1-\gamma}-1}{1-\gamma} + \log(\frac{M_{t}}{P_{t}}) - \alpha L_{t} \right) \mathrm{d}t \\ &s.t. \quad C_{t} = \left( \int_{0}^{1} A_{i,t}^{\frac{1}{\eta}} C_{i,t}^{\frac{\eta-1}{\eta}} \mathrm{d}i \right)^{\frac{\eta}{\eta-1}} \\ &M_{0} + \int_{0}^{\infty} e^{-\int_{0}^{t} R_{s} \mathrm{d}s} \left( w_{t} L_{t} - \ldots \right) \mathrm{d}t = 0 \end{split}$$

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• Optimality Conditions:

$$C_t^{-\gamma} = \alpha P_t / w_t, \quad w_t = \alpha r M_t$$
$$\implies d\ln(C_t) = \gamma^{-1} (d\ln(M_t) - d\ln(P_t))$$

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Firms' Production

$$Y_{i,t} = \frac{1}{Z_{i,t}} L_{i,t}$$

- Marginal cost:  $w_t Z_{i,t}$   $\ln(Z_{i,t}) = \sigma W_{i,t}$ ,  $W_{i,t}$ : Wiener proc.

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denote i's desired price at t

- Desired price = optimal under flexible prices
- Given a price  $p_{i,t}$ , *i*'s instantaneous (approximate) profit losses:

$$\Pi(p_{i,t}, q_{i,t}) - \max_{p} \Pi(p, q_{i,t}) \approx -\frac{B}{2} (p_{i,t} - q_{i,t})^{2}, \qquad B \equiv \eta(\eta - 1)$$

#### Nominal rigidities: general time-dependent frictions

- Price changes arrive according to distribution G with hazard function  $\theta(h)$ 
  - Calvo (1983):  $\theta(h) = \theta$
  - Taylor (1979):  $\theta(h) = \delta_T$  (Dirac delta function centered at some *T*)
- We assume arrivals are *i.i.d.* across firms and counted by  $N_{i,t}$ ,  $\forall i$

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- **Endogenous Attention**: Given  $S_{i,0}$ , design  $\{s_{i,t}\}_{t\geq 0}$  subject to Shannon cost:

$$d\mathbb{I}_{i,t} = \mathbb{I}(Q_{i,t}; S_{i,t}) - \lim_{h \uparrow t} \mathbb{I}(Q_{i,t}; S_{i,h})$$
$$= \lim_{h \uparrow t} \frac{1}{2} \ln \left( \frac{\operatorname{var}(q_{i,t}|S_{i,h})}{\operatorname{var}(q_{i,t}|S_{i,t})} \right)$$

(if jointly Gaussian)

### Setup: Firms' Problems

• Firm *i* chooses pricing and attention policies to minimize lifetime losses:

$$\min_{\{s_{i,t}, \bar{p}_{i,t}: t \ge 0\}} \mathbb{E} \left[ \int_0^\infty e^{-rt} \left( \underbrace{\frac{B}{2} (p_{i,t} - q_{i,t})^2 dt}_{\text{loss from mis-pricing}} + \underbrace{\omega d\mathbb{I}_{i,t}}_{\text{cost of information}} \right) \middle| S_{i,0} \right]$$

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s.t.  $dp_{i,t} = (\tilde{p}_{i,t} - p_{i,t}) dN_{i,t}$   
 $dN_{i,t} \sim \theta(.)$   
 $S_{i,0}, p_{i,0}$  given.

### Firms' Problem: A Conceptual Decomposition of Price Gaps

• Firm *i*'s losses from mis-pricing are a function of the gap  $q_{i,t} - p_{i,t}$ :

$$\underbrace{q_{i,t} - p_{i,t}}_{\text{true price gap}} = \underbrace{\mathbb{E}[q_{i,t}|S_{i,t}] - p_{i,t}}_{x_{i,t} \equiv \text{perceived price gap}} + \underbrace{q_{i,t} - \mathbb{E}[q_{i,t}|S_{i,t}]}_{b_{i,t} \equiv \text{belief gap}}$$

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- $x_{i,t}$  captures nominal rigidities (zero without nominal rigidities)
- $b_{i,t}$  captures information frictions (zero under FIRE)
- Firm's perceived losses at instant t are

$$\mathbb{E}[(q_{i,t} - p_{i,t})^2 | S_{i,t}] = x_{i,t}^2 + \underbrace{\operatorname{var}(q_{i,t} | S_{i,t})}_{\equiv U_{i,t}}$$

with two state variables, which separate firms' attention and pricing decisions:

- **pricing policy**: control  $x_{i,t}$  given information set—here, optimal to reset to zero when price change opportunity arrives
- **information policy**: control  $U_{i,t}$  given  $\theta(.)$  shock and past signals

# **Firms' Information Acquisition**

# **Optimal Dynamic Information Policy**

#### **Theorem 1. Optimal Dynamic Information Policy**

- A firm only acquires information when it changes its price.
- When it does so, it acquires enough information to attain a Gaussian posterior uncertainty of U<sup>\*</sup> that is independent of its state and the unique solution to:

$$\underbrace{\frac{1}{U^*} - \mathbb{E}^h \left[ e^{-rh} \frac{1}{U^* + \sigma^2 h} \right]}_{\text{MC}} = \underbrace{\frac{B}{\omega r} \left( 1 - \mathbb{E}^h [e^{-rh}] \right)}_{\text{MB}}$$

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• Taylor with price spell duration *T*:

$$\lim_{r \to 0} U^* = -\frac{\sigma^2 T}{2} + \sqrt{\left(\frac{\sigma^2 T}{2}\right)^2 + \omega \frac{\sigma^2}{B}}$$

### Corollary. Uncertainty and Time Since Changing Price

Consider a firm i at time t that changed its price h periods ago. The firm's uncertainty about its optimal price follows:

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- Next: What are the implications of this selection for monetary non-neutrality?

# **Implications for Monetary Non-Neutrality**

### Step I: Characterizing Lifetime Output Gaps Y(S) – Full Info

- Money supply increases  $\delta$  percent at t = 0
- Firms' nominal wage increase immediately to  $\delta$  forever



### Step I: Characterizing Lifetime Output Gaps Y(S) – Full Info

- Consider a firm i who last changed its price at  $-h_i$  and gets to reset at  $h'_i$
- With full information, price jumps at new  $w = \delta$  at the first opportunity



### Step I: Characterizing Lifetime Output Gaps Y(S) – Full Info

- Firm *i*'s contribution to output is its duration since shock  $(h'_i)$  times  $\delta$
- Aggregate contribution to output is average duration times  $\delta$



### Step I: Characterizing Lifetime Output Gaps Y(S) – Incomplete Info

- Firms' nominal wage increase immediately to  $\delta$  forever
- Firm *i*: price no longer jumps to  $w = \delta$  at first price change (info. frictions)



### Step I: Characterizing Lifetime Output Gaps Y(S) – Incomplete Info

- Instead, at every new price change, it gets closer to the new  $w = \delta$
- At every price change, the size of the jump depends on the spell duration



### Step I: Characterizing Lifetime Output Gaps Y(S) – Incomplete Info

- Firm *i*'s average contribution to output is now the sum of all these rectangles
- Aggregate non-neutrality is the sum over all firms



### Step II: Constructing the Cumulative Impulse Response (CIR)

For a permissible  $\mathscr{F}$ , the CIRs to observed ( $\mathscr{M}^{x}(\mathscr{F})$ ) and unobserved shock ( $\mathscr{M}^{x}(\mathscr{F})$ ) are:

$$\mathcal{M}^{x}(\mathcal{F}) \equiv \frac{\partial \mathcal{M}(\mathcal{F})}{\partial \mathbb{E}_{\mathcal{F}}[y^{x}]} = \bar{D}$$
$$\mathcal{M}^{b}(\mathcal{F}) \equiv \frac{\partial \mathcal{M}(\mathcal{F})}{\partial \mathbb{E}_{\mathcal{F}}[y^{b}]} = \bar{D} + \bar{D}_{0} \frac{1 - \bar{\kappa}}{\bar{\kappa}_{0}}$$

(observed shock)

(unobserved shock)

- $\bar{D}$  is the average pricing duration of population
- $\bar{D}_0$  is the conditional expected spell duration for price-adjusters
- $\bar{\kappa}_0$  is the expected Kalman gain at the next price reset opportunity for the price-adjusters
- $\bar{\kappa}$  is the average across all firms of the expected Kalman gain when they next reset their prices

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#### **Theorem 2. CIR Characterization**

For any permissible initial distribution  $\mathscr{F} \in \Delta(\mathbb{R}^3)$ , we have that:

$$\mathcal{M}^{b}(\mathcal{F}) - \mathcal{M}^{x}(\mathcal{F}) = \bar{D}_{0} \frac{1 - \bar{\kappa}}{\bar{\kappa}_{0}} = \frac{\boldsymbol{U}^{*}}{\boldsymbol{\sigma}^{2}}$$

### The Roles of Incomplete Information and Selection

• Selection in information acquisition dampens monetary non-neutrality:

#### **Corollary 3. Comparison to Exogenous Information**

With exogenous information:

$$\mathcal{M}^{\text{exo}}(\mathscr{F}) \equiv \frac{\partial \mathcal{M}(\mathscr{F})}{\partial \mathbb{E}_{\mathscr{F}}[y^b]} = \bar{D} + \frac{\bar{U}}{\sigma^2}, \quad \bar{U} = \mathbb{E}_f[U_{i,t}]$$

So the difference between the normalized CIRs to permanent and unobserved monetary shocks under exogenous uncertainty and endogenous uncertainty is:

$$\Delta^{\text{Select}} \equiv \mathcal{M}^{\text{exo}} - \mathcal{M}^{b} = \frac{\bar{U} - U^{*}}{\sigma^{2}} > 0$$

# **Identification of Monetary Non-Neutrality**

$$\mathcal{M}^b = \bar{D} + \frac{U^*}{\sigma^2}$$

- Key finding in state- and time-dependent pricing models with full information: CIR can be identified from data on price changes
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$$H(\Delta p) = \int_0^\infty \Phi\left(\frac{\Delta p}{\sigma\sqrt{h}}\right) \mathrm{d}G(h)$$

where  $\Phi$  is the standard normal CDF.

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⇒ Not enough with incomplete info: surveys are **necessary!** 

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### Data on Uncertainty and Durations Are Sufficient for Identification

• Let *l* be the cross-sectional density of firms' uncertainty



• Key implication: if we have data on firms' pricing durations and uncertainties, we can identify the CIR!

# Using Survey Data to Quantify the Model

# The Survey Data

Survey of firm managers in New Zealand, implemented between 2017Q4 and 2018Q2 (see Coibion, Gorodnichenko, and Kumar, 2018; Coibion, Gorodnichenko, Kumar, and Ryngaert, 2021)

#### Survey question on distribution of beliefs about own price:

"If your firm was free to change its price (i.e. suppose there was no cost to renegotiating contracts with clients, no costs of reprinting catalogues, etc...) today, what probability would you assign to each of the following categories of possible price changes the firm would make? Please provide a percentage answer."

#### Survey question on time since last price change:

"When did your firm last change its price (in months) and by how much (in % change)?"

( Estimation Procedure ) ( Model Prediction: RI vs. Noisy Info. ) ( Model Prediction: Regression



### The Quantitative Importance of Uncertainty and Selection Counterfactuals



# Conclusion

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- We study how measured beliefs can be used to identify monetary non-neutrality
- Optimal dynamic information policies imply selection in information acquisition
- Data on the cross-sectional distributions of uncertainty and pricing durations are both necessary and sufficient to identify monetary non-neutrality
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- Informational frictions approximately double monetary non-neutrality, but models with exogenous information would overstate it by approximately 50%
- Zooming out: implications for survey data ...
  - While random sampling in surveys is important for unbiased estimation of population averages, we provide an example where the average overestimates information frictions
  - Measuring the relevant expectations for aggregate outcomes requires theoretical investigation of whose expectations matter for which outcomes and when those expectations should be measured



# **Comparison to Exogenous (Noisy) Information**



#### Remark

With noisy information, information acquisition is smooth over time and happens at some constant rate of  $\bar{\lambda}$ :

$$\mathrm{d}U_{i,t} = \sigma^2 - \bar{\lambda}U_{i,t}$$

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Thus, belief updating is smooth as well:

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This is equivalent to the model with infinitely convex cost of attention:

$$C(\mathrm{d}\mathbb{I}) = \begin{cases} 0 & \mathrm{d}\mathbb{I} \le \bar{\lambda} \mathrm{d}t \\ \infty & \mathrm{d}\mathbb{I} > \bar{\lambda} \mathrm{d}t \end{cases}$$

### Theory to Estimation: A Practical Estimation Scheme for the CIR

1 The uncertainty of price-setters is given by the mode of the uncertainty distribution:

 $\hat{U}^* = \mathsf{mode}_{\hat{l}}[U]$ 

② The model-implied uncertainty distribution is:

$$l^{M}(z;\sigma^{2}) = \mathbb{I}[z \ge \hat{U}^{*}] \frac{1}{\sigma^{2}} \hat{f}\left(\frac{z - \hat{U}^{*}}{\sigma^{2}}\right)$$

So we can estimate  $\sigma^2$  according to:

$$\hat{\sigma}^2 \in \operatorname{arg\,min} \int_{\hat{U}^*}^{\infty} \left( \hat{l}(z) - l^M(z;\sigma^2) \right)^2 \mathrm{d}z$$

③ Now we have estimates of  $\overline{D}$ ,  $U^*$ , and  $\sigma^2$ , pinning down the CIR  $\mathcal{M}$ 



# Aggregation: Distribution of Uncertainty



#### Proposition. Time invariant distribution of subjective uncertainty: *l*(*U*)

• with exogenous information, it is a mass point at

$$\bar{U} = \frac{\sigma^2}{\bar{\lambda}}$$

or inherits the distribution(s) of in  $\sigma^2$  or  $\bar{\lambda}$  (ex-ante heterogeneity)

# Aggregation: Distribution of Uncertainty



#### Proposition. Time invariant distribution of subjective uncertainty: *l(U)*

• with exogenous information, it is a mass point at

$$\bar{U} = \frac{\sigma^2}{\bar{\lambda}}$$

or inherits the distribution(s) of in  $\sigma^2$  or  $\overline{\lambda}$  (ex-ante heterogeneity)

• with *endogenous information*, it inherits the distribution of ongoing spells *f* (*ex-post* heterogeneity):

$$l(U) = \begin{cases} 0, & U < U^*, \\ \frac{1}{\sigma^2} f\left(\frac{U - U^*}{\sigma^2}\right), & U \ge U^*. \end{cases}$$

# Aggregation: Distribution of Uncertainty

### Proposition. Time invariant distribution of subjective uncertainty: *l*(*U*)

• e.g., with the linear cost and Calvo, it is an exponential distribution: Density



# Prediction: Uncertainty and Time Since Last Price Change



Subjective uncertainty as a function of time since last price change:

• Endogenous information:

uncertaitny grows linearly with time since last price change

• Exogenous information:

uncertatiny is not related to time since last price change

Survey data on firms' subjective uncertainty and time since last price change Coibion, Gorodnichenko, and Kumar (2018); Coibion, Gorodnichenko, Kumar, and Ryngaert (2021)

# **Prediction: Uncertainty and Time Since Last Price Change**



	(1)	(2)	(3)
Dependent variable: Subjective uncertainty about firms' ideal price changes			
Dummy for price changes in the last 3 months	0.0495		
	(0.0862)		
Dummy for price changes in the last 6 months		0.0306	
		(0.0850)	
Dummy for price changes in the last 12 months			-0.643***
			(0.151)
Observations	467	467	467
R-squared	0.114	0.114	0.153
Industry, Firm-level, and Manager Controls	Yes	Yes	Yes

# Counterfactuals: Price Stickiness, Volatility, and Non-Neutrality (Pack)



Microeconomic volatility dampens monetary non-neutrality

2 Price stickiness increases non-neutrality, but by 20% less than under full information

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