

# The Cyclicalities of Wages, Job Duration and Match Quality

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## Abstract

The paper estimates the cyclicalities of real wages differentiating between continuing workers, hires from non-employment and from employment, controlling for various possible sources of composition bias. The wages of hires from employment and to a lesser extent the wages of hires from non-employment are found to be noticeably countercyclical. A proposed explanation is cyclical selection on match quality. The countercyclicality of match quality is supported by the observed relationship between risk of separation and initial conditions. The paper presents a stochastic directed search model with idiosyncratic match quality that incorporates the proposed mechanism.

## 1 Introduction

Understanding the behavior of wages of new hires over the business cycle is crucial for explaining employment fluctuations, as discussed in Pissarides (2009). Since the volumes of hires from employment and non-employment tend to be similar, and it is not obvious that there are no differences in their cyclical properties, it is worthwhile to investigate the two types of hires separately, which much of the earlier literature on wages cyclicalities fails to do, with only recent exceptions such as Getler, Huckfeldt and Trigari (2016). This paper presents new results on the differential cyclicalities of wages for two types of hires and a model with a mechanism that explains the observed differences.

To investigate the cyclical nature of wages, I use a matched employer-employee administrative dataset from Germany that allows both the differentiation between two types of hires and controlling for possible sources of composition bias. The common concern about the validity of studies of wage cyclicality is composition bias with respect to unobserved worker heterogeneity, as discussed in Bils (1985) or Solon, Barsky and Parker (1994). The dataset allows me to remove composition bias not only with respect to unobserved worker heterogeneity but also with respect to firm and occupational composition of jobs.

The wages of new hires turn out to be countercyclical. The countercyclical nature is more pronounced for hires from employment. The unexpected results call for an explanation.

I propose an explanation based on match selection over the business cycle. A worker-firm contact leads to job creation only if idiosyncratic match productivity is high enough. When aggregate productivity is low, the threshold for match quality is higher, which pushes up the wages of new hires if they depend on idiosyncratic productivity.

The proposed mechanism finds additional empirical support. I investigate the relationship between job duration, a standard proxy for match quality, and initial conditions. Matches are at decreased risk of separation into non-employment and tend to last longer if they are started in a period of higher unemployment, which suggests that their idiosyncratic productivity is countercyclical.

Finally, I build a stochastic directed search model in the vein of Menzio and Shi (2010, 2011). The model incorporates the proposed mechanism. Later, I will calibrate the model.

## 2 Related Literature

The main empirical part of this paper belongs to the vast literature on the cyclical properties of real wages. In this section, I discuss how the contributions made in the paper relate to previous empirical findings.

The relationship between the risk of separation and initial aggregate conditions was investigated in few papers that used American data, starting with Bowlus (1995). Recent papers present findings suggestive of countercyclical match quality, in contrast to earlier research that suggested countercyclical match quality, motivating Barlevy (2001).

## 2.1 Cyclicalities of Wages

How do real wages react to the business cycle conditions? This simple question motivated a large body of research and is still not fully settled, despite renewed interest in the cyclicalities of wages of new hires motivated the failures of the canonical search and matching model, as discussed in Pissarides (2009).

Up to the early 1990s, the consensus, based on studies using aggregate data, was that real wages in the US were acyclical or, at best, weakly procyclical. As Stockman (1983) surmised, the composition of the labor force changes over the cycle. More pronounced procyclicalities of hours and employment of low-wage workers induces a countercyclical bias in an aggregate measure of wages. An opposite procyclical effect was identified in Chirinko (1980) as arising from high cyclical sensitive of high-wage industries such as durables manufacturing and construction.

Starting with Bils (1985) and Solon, Barsky and Parker (1994), the use of individual level data shattered the previous consensus. The wages were found to be procyclical.

Two recent papers, Haefke, Sonntag and van Rens (2013) and Gertler, Huckfeldt and Trigari (2016), differentiate between hires from employment and hires from non-employment. Haefke, Sonntag and van Rens (2013) find the elasticity of wages with respect to labor productivity to be higher for hires from non-employment than for job stayers, and even higher for hires from employment. However, they use cross-sectional data from the Current Population Survey outgoing rotation groups which do not allow for controlling for unobserved worker heterogeneity. Gertler, Huckfeldt and Trigari (2016) find wages of job stayers in data from the Survey of Income and Program Participation to be slightly procyclical, wages of hires from non-employment to be acyclical and wages of hires from employment to be procyclical. They argue that wages might be set by staggered contracting and relatively rigid and excess cyclicalities of wages of job changers might reflect procyclical match quality for hires from employment.

The use of administrative datasets made it possible to control for various potential sources of composition bias. Carneiro, Guimaraes and Portugal (2012) and Martins, Solon and Thomas (2012) use Portuguese Quadros de Pessoal data that match employees with employers. In the first paper, the cyclicalities of wages is estimated with controls for worker,

job and occupation fixed effects and find wages of new hires to be more procyclical than wages of job stayers. The second paper concentrates on hiring wages for a set of entry jobs which are found to be quite procyclical. Due to limitations of the dataset, these papers cannot differentiate between hires from employment and from non-employment.

Stueber (2017) used the same source of data as my paper, the employment biographies generated by the German social security system, but for the period 1977-2009. The wages of newly hired workers were found to be no more procyclical, controlling for worker and employer-occupation fixed effects.

## 2.2 Match Quality

Match quality is not directly observable. A traditional proxy is job duration - a better match should last longer.

Bowlus (1995) found that the initial unemployment rate had a negative effect on job duration, suggesting the procyclicality of match quality and motivating Barlevy (2002) to formulate a theory of sullyng recessions. The results were confirmed in del Rio (2012).

However, two recent papers presented different results on the relationship between the initial unemployment rate and job duration in the US. Baydur and Mukoyama (2015) estimated the higher initial unemployment rate to increase the probability of job-to-job transition but not the probability of separation into nonemployment. According to Baley and Figueiredo (2017) the initial higher unemployment rate increases the probability of a job-to-job transition but decreases the probability of separation into nonemployment. These results together with mine raise the possibility that match quality is countercyclical both in Germany and the US.

## 3 Data

The paper uses the LIAB Longitudinal Model 1993-2010 (LIAB LM 9310), a linked employer-employee dataset provided by the Research Data Centre (FDZ) of the Federal Employment Agency (BA) at the Institute for Employment Research (IAB). In this section, I describe its origin, structure, contents, sample selection and two significant drawbacks: lack of infor-

mation on hours worked and censoring.

Neither of the drawbacks is critical. Due to institutional features of German labor market and data generation, earnings of full-time workers are roughly equivalent to wages when controls for firm effects are included. I discuss how estimates obtained in Card, Heining and Kline (2013) are used to check for the robustness of results with respect to censoring problems

A detailed description of the dataset is provided in Klosterhuber, Heining and Seth (2014).

### **3.1 Generation of Data**

The LIAB links information on establishments from the annual waves of the IAB Establishment Panel and on workers from the process-generated data of the BA.

The establishments are the 2000-2008 panel cases of the IAB Establishment Panel. The Establishment Panel is an annual survey which sample is drawn from the population of all establishments with employees covered by social security. The sample is stratified with respect to industry, size and federal state.

The workers are individuals who were employed in one of the selected establishments for at least one day between 1999 and 2009. Their employment biographies for the period 1993-2010 are taken from the Integrated Employment Biographies (IEB) database of the IAB.

The IEB integrates the Employee History (BeH) data with information about receipt of social benefits, registration as a job seeker and participation in an employment or training measure provided by social security agencies.

The BeH data are generated by a mandatory notification procedure for health, pension and unemployment insurance, started in 1973 for West Germany and in 1991 for East Germany. Employers are obliged to submit notifications to the social security agencies whenever the terms of employment change and at the end of a year. The BeH covers all workers and apprentices unless they are exempt from social security contributions (civil service, army, judiciary, self-employed, regular students). Starting in 1999, marginal part-time employment and unpaid family work are subject to notifications.

The legal sanctions for misreporting and checks performed by the social security agencies ensure high reliability of the BeH data.

### **3.2 Structure of Data**

The employers submit notifications whenever the terms of employment change and at the end of a year. The notifications are the original observations from which the BeH is generated. However, if original observations overlap for an individual, they are replaced with artificial observations with new dates ("splitting"). Consequently, the observation periods ("spells") are either completely parallel or non-overlapping.

### **3.3 Content of Data**

An observation contains worker and establishment identifiers, the start and end dates for original and splitted observations, sex, education, working hours (full-time or part-time), employment status, daily earnings and other information.

I construct daily earnings for an employee-employer pair by adding up daily earnings in all parallel episodes for a pair and dividing them by the length of the considered period. Earnings are deflated by the CPI.

### **3.4 Sample Selection**

The sample is restricted to the spells of employment in West German establishments that are the 2000-2008 panel cases of the IAB Establishment Panel. The workers considered are males aged 20-60. The age and sex restriction is adopted for comparability with earlier studies.

### **3.5 Earnings or Wages?**

The information on working hours is partial. It is observed whether an employee works full- or part-time. The distinction is made by comparing the employee's contracted hours and the usual working hours in the establishment.

In the main specification, I use only full-time non-trainee workers. When firm fixed effects are included, their earnings are roughly equivalent to wages.

### 3.6 Censoring

The observations with daily earnings above the contribution assessment ceiling (Beitragsbemessungsgrenze) are topcoded. More than 10% of observations are affected.

The main results are based on the uncensored observations. To check for robustness with respect to censoring, I use estimates of worker and establishment fixed effects from Card, Heining, Kline (2013) are used to control for worker and establishment heterogeneity in a Tobit regression.

## 4 Empirical Results

In this section, I discuss the specification for the estimation of the cyclicity of wages and the results, and the specification for the estimation of the relationship between risk of separation and initial conditions.

### 4.1 Wages: Specification

The specification is similar to Gertler, Huckfeldt and Trigari (2016). Data are at monthly frequency. Let  $w_{it}$  denote the real wage paid in period  $t$  to individual  $i$ . The wage equation is

$$\log w_{it} = \pi u_t + \pi_E D_E(i, t) u_t + \pi_U D_U(i, t) u_t + \gamma'_x \mathbf{x}_{it} + \epsilon_{it} \quad (1)$$

where  $u_t$  is the unemployment rate,  $D_E(i, t)$  is an indicator variable that takes value one for new hires from employment,  $D_U(i, t)$  is an indicator variable that takes value one for new hires from unemployment, vector  $\mathbf{x}_{it}$  contains a time trend (calendar-month dummies and a quadratic polynomial in time) and indicators  $D_E(i, t)$ ,  $D_U(i, t)$  in all specifications, and, additionally, controls for observable worker heterogeneity (an education-specific cubic polynomial in age, and a cubic polynomial in tenure when applicable), worker fixed effects, fixed effects for occupations, and establishment fixed effects.

The coefficients of interest are  $\pi$ , the semielasticity of wages with respect to the unemployment rate  $u_t$ , the incremental effect for hires from employment,  $\pi_E$ , and the incremental effect for hires from unemployment,  $\pi_U$ .

Hires from employment are identified as workers that started their current job no more than 14 days after the end of their previous employment and without registering with the BA as an unemployed or a jobseeker, while hires from unemployment are identified as workers that started their current job more than 14 days after the end of their previous employment or after registering with the BA. The conclusions are robust to changing the cutoff for differentiation between hires from employment and unemployment to 31 days.

In the main specification, equation (1) is estimated for uncensored observations and full-time non-trainee workers. I also estimate (1) using all observations and a generalized Tobit regression. Due to the size of the sample, it is not feasible to fixed effects for this estimation. Instead, I use worker and establishment fixed effects from Card, Heining and Kline (2013) as controls. Additionally, I estimate (1) adding fixed effects for working hours and employment status [to be added later].

## 4.2 Wages: Results

The main results are in column (7) of Table 1. The cyclicality of wages of hires from employment is captured as  $\hat{\pi} + \hat{\pi}_E$  and is positive, indicating countercyclicality. The countercyclicality of wages of hires from unemployment is present but less pronounced.

Table 2 presents results for the sample restricted to the observation for which worker and establishment effects estimated by Card, Heining, Kline (2013) are available. The results of a Tobit regression for the sample including censored observations are in Column (3) and can be compared to the results of an OLS regressions with Card, Heining and Kline (2013) effects in Column (1) and with worker and establishment fixed in Column (2) excluding censored observations. The results are quantitatively similar.

[the results with fixed effects for working hours status and employment status are to be added later, but are comparable]



Table 1: Main Results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\hat{\pi}$	0.603**** (0.056)	0.335**** (0.056)	-0.054 (0.043)	-0.049 (0.043)	0.397**** (0.052)	-0.054 (0.044)	-0.049 (0.043)
$\hat{\pi}_E$	1.477* (0.691)	1.488* (0.749)	0.736** (0.221)	0.742** (0.236)	1.559**** (0.380)	0.675**** (0.161)	0.687*** (0.178)
$\hat{\pi}_U$	3.295** (1.124)	2.793** (1.055)	0.414* (0.191)	0.456* (0.192)	1.300** (0.492)	0.346* (0.162)	0.394* (0.166)
adj. R-sq	0.015	0.225	0.867	0.866	0.399	0.868	0.867
Worker Observables	No	Yes	Yes	Yes	No	Yes	Yes
Worker FE	No	No	Yes	Yes	No	Yes	Yes
Occupation FE	No	No	No	Yes	No	No	Yes
Est. FE	No	No	No	No	Yes	Yes	Yes

*Notes:*

+  $p < .1$ , \*  $p < .05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$  \*\*\*\*  $p < .0001$ ; time-clustered standard errors in parentheses; numbers of observations, establishments and workers in estimation samples are in Table 3 in Appendix.

Table 2: Results for Uncensored and Censored Observations

	(1)	(2)	(3)
$\hat{\pi}$	-0.121** (0.043)	-0.097* (0.042)	-0.228**** (0.042)
$\hat{\pi}_E$	0.922*** (0.250)	0.706**** (0.161)	0.916**** (0.215)
$\hat{\pi}_U$	0.389 (0.327)	0.338* (0.169)	0.306 (0.291)
adj. R-sq	0.765	0.867	
Worker Observables	Yes	Yes	Yes
Worker FE	No	Yes	Yes
Worker CHK Effect	Yes	No	No
Occupation FE	No	No	No
Est. FE	No	Yes	No
Est. CHK Effect	Yes	No	Yes
Regression	OLS	OLS	Tobit

*Notes:*

+  $p < .1$ , \*  $p < .05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$  \*\*\*\*  $p < .0001$ ;  
time-clustered standard errors in parentheses;  
numbers of observations, establishments and workers in estimation samples are in Table 4 in Appendix.

### 4.3 Job Duration: Specification

[to be added later]

### 4.4 Job Duration: Results

[to be added later, they are robust, and statistically and economically significant]

## 5 Model

In this section, I outline a model based on the stochastic search model of Menzio and Shi (2010,2011) that replaces indexing by promised utility with indexing by a promised Nash bargaining parameter.

The existence of an equilibrium is proved with the use of the Knaster-Tarski theorem.

### 5.1 Model Outline

There is a continuum of risk-neutral workers with measure one. They maximize the expected sum of periodical incomes, discounting with factor  $\beta \in (0, 1)$ . The unemployed, which measure is denoted as  $u$ , receive benefit  $b$ .

There is a continuum of firms with positive measure. Each firm turns one unit of labor into  $r(y, z)$  units of output. The aggregate productivity,  $y$ , is the same for all firms, with values in the set  $Y = \{y_1, y_2, \dots, y_{N_Y}\}$ , where  $y_1 < y_2 < \dots < y_{N_Y}$  and  $N_Y \geq 2$ . The aggregate productivity is updated to  $\hat{y}$  at the beginning of the next period with probability  $f_Y(y, \hat{y})$ , where  $f_Y : Y^2 \rightarrow [0, 1]$ . The idiosyncratic productivity  $z$  belongs to the set  $Z = \{z_1, z_2, \dots, z_{N_Z}\}$ , where  $z_1 < z_2 < \dots < z_{N_Z}$  and  $N_Z \geq 2$ . It is fixed for a match and drawn from the probability distribution  $f_Z : Z \rightarrow [0, 1]$  when a worker and a firm meet.

The labor market is a submarkets indexed by the share of match surplus  $\tau \in [0, 1]$  that the firms offer to the workers. If the worker and the firm met in submarket  $\tau$  and drew idiosyncratic productivity  $z$ , then in period with aggregate productivity  $y$  the wage  $w(\tau, y, z)$  is determined to the worker's surplus is equal to fraction  $\tau$  of the match surplus. If the value of match to the firm is denoted as  $J(\tau, y, z)$ , the value of match to the worker as  $V(\tau, y, z)$ , the value of unemployment as  $U(y)$ , then the match surplus is  $S(\tau, y, z) = J(\tau, y, z) + V(\tau, y, z) - U(y)$ . The contract specifies that  $V(\tau, y, z) - U(y) = \tau S(\tau, y, z)$ .

The ratio of the number of vacancies created by firms to the number of workers looking for jobs, the tightness of submarket  $\tau$ , is denoted as  $\theta(\tau, y)$  and determined in the equilibrium in a manner specified later by choices of firms that take into account the cost of opening a vacancy,  $k > 0$ , the expected benefit conditional on meeting a worker,  $(1 - \tau)\tilde{S}(\tau, y)$ , where  $\tilde{S}(\tau, y) = \int \mathbb{1}\{S(\tau, y, z) > c\}S(\tau, y, z)f_Z(z)dz$ , and the probability of meeting a worker,

$q(\theta(\tau, y))$ , where  $q : \mathbb{R}_+ \rightarrow [0, 1]$  is a strictly decreasing function with  $q(0) = 1$ . A worker meets a vacant job with probability  $p(\theta(\tau, y))$ , where  $p : \mathbb{R}_+ \rightarrow [0, 1]$  is a strictly increasing function with  $p(0) = 0$ . The functions  $q, p$  satisfy the relationship  $p(\theta) = \theta q(\theta)$ . When a firm and a worker meet, the idiosyncratic productivity is drawn, and the match is established if the surplus net of job creation cost  $c \geq 0$ .

Each period consists of four stages: separation, vacancy creation and search, matching and, finally, production. In the separation stage, an employed worker moves into unemployment with probability  $d$  that is specified later and no smaller than the exogenous separation probability  $\delta \in (0, 1)$ . In the second stage, an unemployed worker can search for a job with probability  $\lambda_U \in (0, 1)$ , an employed worker can search for a job with probability  $\lambda_E \in \{0, \Lambda_E\}$ , where  $\Lambda_E \in (0, 1)$ , which is specified later. The choice of submarket of an employed worker is specified later. At the same time, firm choose how many vacancies they create and in which submarkets they locate vacancies. All agents treat  $\theta(\tau, y)$  as a parameter. In the matching stage, the workers and the firms meet, idiosyncratic productivities are drawn and job creation decisions are made. In the production stage, an unemployed worker receives  $b \in (0, r(y_{N_Y}, z_{N_Z}))$ , an employed worker in a match characterized by  $\tau$  and  $z$  produces  $r(y, z)$  and receives  $w(\tau, y, z)$ , when his employer collects  $r(y, z) - w(\tau, y, z)$ .

## 5.2 Vacancy Creation Decisions

A firm chooses the number and location of vacancies. The value of creating a vacancy in submarket  $\tau$  depends on the probability of meeting a worker,  $q(\theta(\tau, y))$ , and the expected value of job to the firm,  $(1 - \tau)\tilde{S}(\tau, y)$ . When this value is higher than the cost of creating a vacancy,  $k > 0$ , the firm creates vacancies in submarket  $\tau$ . When the value is lower than the cost of creating a vacancy, the firm creates no vacancies in submarket  $\tau$ . The ratio of vacancies to job searchers, the tightness  $\theta$ , adjusts to bring about the equilibrium .

The tightness  $\theta(\tau, y) \geq 0$  satisfies

$$k \geq q(\theta(\tau, y))(1 - \tau)\tilde{S}(\tau, y)$$

and in the equilibrium is determined as

$$\theta(\tau, y) = \begin{cases} q^{-1}(k/(1-\tau)\tilde{S}(\tau, y)), & \text{if } (1-\tau)\tilde{S}(\tau, y) \geq k \\ 0, & \text{if } (1-\tau)\tilde{S}(\tau, y) < k \end{cases} \quad (2)$$

### 5.3 Value Functions

The value accruing to an unemployed worker in the production stage is

$$U(y) = b + \beta \mathbb{E} \left[ U(\hat{y}) + \lambda_{UP}(\theta(\hat{\tau}_U, \hat{y}))(\hat{\tau}_U \tilde{S}(\hat{\tau}_U, \hat{y}) - U(\hat{y})) \right] \quad (3)$$

where  $\hat{\tau}_U$  is forced to be equal to 0 but could be a result of worker's maximization in the general case.

The value accruing to an employed worker in the production stage is

$$\begin{aligned} V(\tau, y, z) = & w(\tau, y, z) + \\ & + \beta \mathbb{E} \left[ \hat{d}U(\hat{y}) + (1 - \hat{d}) \left( V(\tau, \hat{y}, z) + \hat{\lambda}_{EP}(\theta(\hat{\tau}_E, \hat{y}))(\hat{\tau}_E \tilde{S}(\hat{\tau}_E, \hat{y}) - V(\tau, \hat{y}, z)) \right) \right] \end{aligned} \quad (4)$$

where  $\hat{\tau}_E, \hat{d}, \hat{\lambda}_E$  are functions of  $(\hat{y}, z)$  chosen by the firm when the surplus is maximized but could be a result of worker's maximization in the general case. The value can be rewritten as

$$\begin{aligned} V(\tau, y, z) = & w(\tau, z, y) + \\ & + \beta \mathbb{E} \left[ \hat{d}U(\hat{y}) + (1 - \hat{d}) \left( \tau S(\tau, \hat{y}, z) + \hat{\lambda}_{EP}(\theta(\hat{\tau}_E, \hat{y}))(\hat{\tau}_E \tilde{S}(\hat{\tau}_E, \hat{y}) - \tau S(\tau, \hat{y}, z)) \right) \right]. \end{aligned} \quad (5)$$

The value accruing to a firm in the production stage

$$J(\tau, y, z) = r(y, z) - w(\tau, y, z) + \beta \mathbb{E} \left[ (1 - \hat{d})(1 - \hat{\lambda}_{EP}(\theta(\hat{\tau}_E, \hat{y})))J(\tau, \hat{y}, z) \right]. \quad (6)$$

The surplus of a match,  $S(\tau, z, y) = J(\tau, z, y) + V(\tau, z, y) - U(y)$ , can be rewritten as

$$\begin{aligned} S(\tau, y, z) = & r(y, z) - b \\ & + \beta \mathbb{E}(1 - \hat{d}) \left[ S(\tau, \hat{y}, z) + \hat{\lambda}_{EP}(\theta(\hat{\tau}_E, \hat{y}))(\hat{\tau}_E \tilde{S}(\hat{\tau}_E, \hat{y}) - S(\tau, \hat{y}, z)) \right] \end{aligned} \quad (7)$$

if  $\hat{\tau}_U = 0$ .

The surplus maximization means that

$$\begin{aligned} S(\tau, y, z) = & r(y, z) - b \\ & + \beta \max_{\hat{\tau}_E, \hat{d}, \hat{\lambda}_E} \mathbb{E}(1 - \hat{d}) \left[ S(\tau, \hat{y}, z) + \hat{\lambda}_{EP}(\theta(\hat{\tau}_E, \hat{y}))(\hat{\tau}_E \tilde{S}(\hat{\tau}_E, \hat{y}) - S(\tau, \hat{y}, z)) \right] \end{aligned} \quad (8)$$

under the constraints  $\forall_{\hat{y}} \hat{d}(\tau, \hat{y}, z) \in [\delta, 1], \hat{\lambda}_E(\tau, \hat{y}, z) \in \{0, \Lambda_E\}$ .

## 5.4 Equilibrium

An equilibrium of the model consists of a surplus function  $S : [0, 1] \times Y \times Z \rightarrow \mathbb{R}$ , policy functions  $\hat{\tau}_E : [0, 1] \times Y \times Z \rightarrow [0, 1]$ ,  $\hat{d} : [0, 1] \times Y \times Z \rightarrow [\delta, 1]$ ,  $\hat{\lambda}_E : [0, 1] \times Y \times Z \rightarrow \{0, \Lambda_E\}$ , and the tightness function  $\theta : [0, 1] \times Y \rightarrow \mathbb{R}_+$  that satisfy the conditions (1) and (7).

Existence of an equilibrium is established with the use of the Knaster-Tarski theorem. An equilibrium is found as a fixed point of an order-preserving function  $\mathcal{T}$  that acts on a complete lattice  $\mathcal{S}$ .

A set of functions  $\mathcal{S}$  such that  $S \in \mathcal{S}$  iff  $S : [0, 1] \times Z \times X \rightarrow [\underline{S}, \bar{S}]$  where  $\underline{S} = r(z_1, y_1) - b$  and  $\bar{S} = (r(z_{N_Z}, y_{N_Y}) - b) / (1 - \beta)$ , with a partial order defined as  $S_2 \succeq S_1$  for  $S_1, S_2 \in \mathcal{S}$  iff  $\forall_{\tau, z, y} S_2(\tau, z, y) \geq S_1(\tau, z, y)$  is a complete lattice (a partially ordered set in which all subsets have a supremum and an infimum).

The function  $\mathcal{T}$  is defined as

$$\begin{aligned} \mathcal{T}S(\tau, y, z) &= r(y, z) - b \\ &+ \beta \max_{\hat{d}, \hat{\tau}, \hat{\lambda}_E} \mathbb{E}(1 - \hat{d}) \left[ S(\tau, \hat{y}, z) + \hat{\lambda}_E p(\theta(\hat{\tau}_E, \hat{y})) (\hat{\tau}_E \tilde{S}(\hat{\tau}_E, \hat{y}) - S(\tau, \hat{y}, z)) \right] \end{aligned}$$

where  $\forall_{\hat{y}} \hat{d}(\tau, \hat{y}, z) \in [\delta, 1]$ ,  $\hat{\lambda}_E(\tau, \hat{y}, z) \in \{0, \Lambda_E\}$  and  $\theta(\tau, y)$  is determined by (1). It is easily shown that  $\forall_{S \in \mathcal{S}} \bar{S} \succeq \mathcal{T}S \succeq \underline{S}$ , and consequently  $\mathcal{T}S \in \mathcal{S}$ . Since  $\forall_{S_1, S_2 \in \mathcal{S}} S_2 \succeq S_1$   $\mathcal{T}S_2 \succeq \mathcal{T}S_1$ , the function  $\mathcal{T}$  is order-preserving.

The Knaster-Tarski theorem is not constructive. However, the properties of the function  $\mathcal{T}$  can be exploited to devise a binary search-style iterative procedure using  $\underline{S}$  and  $\bar{S}$ .

## 5.5 Myopic Model

The full model have to be solved numerically. However, a simplified version with myopic agents (the discounting factor  $\beta = 0$ ) is analytically solvable. It turns out that wages in the simplified model can have the properties found in the empirical part, and that selection on match quality ensures that matches formed when aggregate productivity is lower are at decreased risk of separation into unemployment.

In this version, I assume that workers choose submarket  $\tau^*$  in which they search. They maximize

$$R(\tau^*, y, f) = p(\theta(\tau^*, y)) (\tau^* \tilde{S}(\tau^*, y, z) - f) \quad (9)$$

where  $f$  is  $U(y)$  for the unemployed and  $V(\tau, z, y)$  for the employed, and consequently search according to a policy function  $\tau^*(y, f)$ .

In the myopic model, the value functions are

$$\begin{aligned} U(y) &= b, \\ &= w(\tau, z, y), \\ J(\tau, z, y) &= r(z, y) - w(\tau, z, y) \end{aligned}$$

and the surplus of a match is

$$S(\tau, z, y) = r(z, y) - b.$$

The Nash bargaining leads to the determination of the job value to a firm and the employment value to a worker as

$$J(\tau, z, y) = (1 - \tau)(r(z, y) - b)$$

and

$$V(\tau, z, y) = \tau r(z, y) + (1 - \tau)b.$$

The expected surplus conditional on meeting a worker and creating a job,  $\tilde{S}(\tau, y)$  is a function of the primitives of the model, and takes form  $\tilde{S}(\tau, y) = (1 - \tau)\tilde{\tilde{S}}(y)$ , where  $\tilde{\tilde{S}}(y)$  increases in  $y$ . The tightness function  $\theta(\tau, y)$  is determined by (1).

The problem (8) can be solved analytically for any functional form of  $p(\theta)$ . When  $p(\theta) = \theta(1 + \theta^\gamma)^{-1/\gamma}$ , it is easily shown with some algebra that  $\tau^*(y, f)\tilde{S}(\tau^*, y, z)$  can decrease in  $y$ , especially if  $\tilde{S}(\tau^*, y, z) - f$  is small (that is, for the employed rather than the unemployed since  $r(z, y) > b$  for wages in continuing matches).

The threshold of idiosyncratic match productivity for job creation is given by the solution to an equation  $\tilde{S}(\tau, y, z) = z$ , which is independent from  $\tau$ . It is easily checked that the threshold is decreasing in  $y$  and some of jobs created when  $y$  is high are characterized by low  $z$ . Consequently, if jobs are exogenously destroyed whenever  $S(\tau, y, z)$  falls below zero, then that matches formed when aggregate productivity is lower are at decreased risk of separation into unemployment.

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## A Appendix

Table 3: Observations, Establishments, Workers

	No. of Observations	No. of Establishments	No. of Workers
(1)	25776203	507543	4555
(2)	25776203	507543	4555
(3)	25758020	489360	4504
(4)	25679448	485942	4490
(5)	25776170	507511	4522
(6)	25758019	489360	4503
(7)	25679447	485942	4489

*Notes:*

row  $(i)$ ,  $i \in \{1, \dots, 7\}$  corresponds to column  $(i)$  of Table 1.

Table 4: Observations, Establishments, Workers

	No. of Observations	No. Establishments	No. of Workers
(1)	24751079	443987	3434
(2)	24736866	429774	3427
(3)	29221597	474070	3439

*Notes:*

row  $(i)$ ,  $i \in \{1, \dots, 3\}$  corresponds to column  $(i)$  of Table 2.