Flooded House or Underwater Mortgage?

The Implications of Climate Change and Adaptation on Housing, Income & Wealth

Yasmine van der Straten *

Updated: January 22, 2024
First Version: March 15, 2023

Abstract

I study the implications of climate change and adaptation on housing and income, and wealth. I embed climate change in a redistributive growth model by introducing exposure of households and firms to extreme weather events, which damage their housing capital and physical capital, respectively. The analysis reveals that climate change is intrinsically redistributive. Low-income workers experience a relatively larger decline in income as climate-related damages reduce their productivity disproportionately. At the same time, the rate at which households with positive savings accumulate wealth rises. Importantly, I show that low-income households who are financially constrained have weaker incentives to adapt to climate change and their failure to reduce vulnerability to climatic impacts exacerbates wealth inequality. While houses exposed to climate risk face a price discount in the market, I demonstrate that the materialization of climate change risk increases house prices as habitat becomes reduced. This general equilibrium effect induces low-income households to allocate a larger fraction of their budget to housing as climatic impacts intensify, translating into a widening of the adaptation gap over time.

***Keywords*—** Climate Change, Climate Change Adaptation, Housing, Financial Assets, Financial Constraints, Extreme Weather Events, Wealth Inequality.

***JEL classification codes*—** E44, G51, Q54.

---

* Amsterdam School of Business, University of Amsterdam; Tinbergen Institute.
I am grateful to Enrico Perotti for many valuable discussions and suggestions that significantly improved this paper. For helpful comments, I thank Rick van der Ploeg, Björn Brügemann, Viral Acharya, Eric Bartelsman, Simone Lenzu, Spyros Terovitis, Yenan Wang, Albert Jan Hummel, Elizabeth Proehl, Bram van der Kroft, Fabian Greimel, Simon J. Toussaint, Daniel Schmidt, Andrea Titton, Robert Lee, and my discusants Pierre Mabille, Achim Hagen, Franck Moraux, Martin Blomhoff Holm, Xingchen Zhu, Deniz Aydin, and Fanny Cartellier. I am thankful to have received the Best Student Paper Award at the CEPR European Conference on Household Finance (2023) and the Best Single-Authored Paper Award at the European Sustainable Finance PhD Workshop (2023). This paper further benefited from discussions with participants of the Woman in Central Banking Research Workshop at Norges Bank, the 9th IWH-Fin-Fire Workshop, the CEPR European Conference on Household Finance 2023, the 12th Portuguese Financial Network Conference, the 1st European Sustainable Finance PhD Workshop, the 39th International Conference of the French Finance Association, the 1st VU Finance PhD Workshop, IPWSD 2023 at Columbia University in the City of New York, the Dutch Economist Day 2022, and participants of the internal research seminar of De Nederlandsche Bank, the Seminars on Environmental Economics and Management of Sustainability as well as the internal seminar of the MInt and Finance group of the University of Amsterdam. All potential errors are my own.

1 Introduction

The significance of climate change adaptation becomes increasingly evident as climatic impacts change intensify across the globe, and mitigation efforts remain inadequate in preventing temperatures from rising above 1.5 degree Celsius (UNFCCC (2022)). While mitigation efforts seek to prevent further climate change, adaptation aims to reduce vulnerability to physical climate risk by taking measures that limit economic losses and damages upon climatic impacts (IPCC (2021)). However, a pressing question remains: can adaptation effectively limit the potentially significant adverse effects of climate change on our economy (Barrage and Nordhaus (2023); Kahn et al. (2021))? In times of rising wealth inequality (Saez and Zucman (2016); Zucman (2019); Blanchet and Martínez-Toledano (2023)), it is essential to understand the differential impacts of climate change and heterogeneity in our ability to respond. How are economic loss and damages distributed? Should we be concerned about feedback effects, given that income and wealth are critical determinants of our ability to adapt? Do financial constraints restrict those most vulnerable to climate impacts when attempting to reduce their vulnerability?

This paper answers the above questions and sheds light on the broader implications of climate change and adaptation on housing, income and wealth. The analysis reveals that climate change is intrinsically redistributive. Wage inequality rises as climate-related damages reduce the productivity of low-income households disproportionately. At the same time, the rate at which households with positive savings accumulate wealth rises, whereas those with debt face a rise in the costs of borrowing. I highlight that adapting to climate change is more challenging for financially constrained, low-income households and the failure to reduce vulnerability to climate change results in a disproportionately large reduction of their wealth once an extreme weather event occurs. This exacerbates wealth inequality. As climatic impacts intensify, housing becomes relatively more important in the consumption bundle of constrained households, inducing constrained households to allocate a larger fraction of their budget on housing. Consequently, the investment in adaptation by constrained households becomes increasingly lower relative to the optimal investment, translating into a widening of the 'adaptation gap' over time. To illustrate these equilibrium effects, I provide a parameterization based on the Netherlands, a country with a long history in flood risk management, which is likely to face an increase in its exposure to flood risk as sea levels rise. I demonstrate the model outcomes for different scenarios of climate change, based on based on a low (RCP 2.6), medium (RCP 4.5), and high (RCP 8.5) greenhouse gas concentration trajectory from IPCC (2013).

I embed climate change and adaptation in a redistributive growth model, which is based on Döttling and Perotti (2017). The economy consists of one region, populated by households and firms. Households have preferences over housing and a non-durable consumption good, which is produced by the firm. Households invest in housing when young. The investment in housing is risky as households are exposed to physical climate risk. Physical risk emerges due to a rise in the probability of being hit by an extreme weather event, such as floods, wildfires, and hurricanes, or changes in climate patterns, such as sea-level rise. The climatic impacts destroy housing capital, leading to a decrease in the expected revenue from selling a house that is exposed to climate risk in the subsequent period. This finding is in line with the empirical literature which consistently show that houses facing climate risk are traded at a discount (see, e.g., Bernstein et al. (2019); Bosker et al. (2019); Baldauf et al. (2020)). However, the materialization of climate risk reduces the supply of inhabitable houses over time, reflecting the premise that certain areas become uninhabitable as a consequence of climate change (Burzyński et al. (2019)).[1] This raises the marginal benefit of owning a house, and hence increases the contemporaneous house prices - an effect which is novel to the literature. Moreover, as the supply of houses shrinks permanently and climate risk rises, this general equilibrium effect becomes stronger over time.

Households differ in terms of their skills, which are exogenously given, and work in the firm when young. High-skilled workers are complementary to intangible capital, whereas low-skilled workers are complementary to physical capital (Krusell et al. (2000); Känzig (2023)). Firms are also exposed to climate risk, as extreme weather events destroy physical capital (Acharya et al. (2022)). This leads to a capital loss, reducing the firm's output and profits. The decline in output causes wages to fall, and due to the decline in income firm to scale back investments in both types of capital. Since physical capital is directly exposed to climate risk, firms use increasingly less physical capital in their production process. Hence, the productivity of low-skilled workers falls disproportionately, leading to a rise in the wage gap and exacerbating income inequality. Climate-related damages further affect the costs of borrowing, as firms scale up their investment in physical capital up to the point where the costs of borrowing are equal to the return on physical capital. While the fall in income reduces consumption and investments, physical capital becomes relatively more scarce over time, which raises its price. This increases the rate at which households with positive savings accumulate wealth, whereas those with debt experience higher costs of debt servicing. The rise in the costs of borrowing further suppresses the value of the firm's equity, reducing the financial wealth of shareholders. Consequently, changes in the price of financial assets become equalizing, while savings form an unequalizing force.[2] Moreover, as house prices rise while income falls, households become more indebted over time, as reflected in rising loan-to-value ratio's. As households' financial positions worsens over time and the exposure to climate risk becomes more severe, this leads to an rise in mortgage default rates.

The paper studies the optimal response of households to rising climate risk, and its implications on wealth inequality. As the analysis is conducted in the context of a closed economy, where climate change is as an exogenous phenomenon, the emphasis is on *adaptation* strategies, rather than *mitigation* measures. Households adapt to climate change by making investments that reduce vulnerability to future climatic impact (IPCC (2007); IPCC (2014);IPCC

[1] It is crucial to highlight that, in the absence of climate risk, the supply of houses were to remain constant over time. Therefore, the evolution of the housing stock specifically pertains to the residual factor, which is the change in supply which is attributable to climate change. Although in reality one could respond to this by constructing new properties, this does not alter the underlying premise that there is a diminishing amount of space available for the overall population to share.

[2] This os in contrast with e.g. Bauluz et al. (2022), Fagereng et al. (2022) and Martínez-Toledano (2020), who show that the current rise in wealth inequality is driven by changes in asset prices, rather than savings.

(2021)). By investing in protective measures, such as installing storm-proof windows or building seawalls and stilts (Fried (2022)), households limit damages to their housing capital, an essential component of household wealth. [3] Hence, adaptation shifts the distribution of idiosyncratic losses to the left, limiting the losses suffered in expectation due to extreme weather event. The investment in adaptation is costly, however, and households trade off the investment costs and the benefits of avoided climate change damage. The investment in adaptation increases in climate risk, as this increases the expected damages to the house, as well as in the house price, as this increases the value that is potentially lost when an extreme weather event occurs. When climate risk is accurately priced in the market (i.e. beliefs about climate risk are correct), households internalize the benefits of their adaptive efforts for future generations. This prevents "tragedy of the horizon" effects (Carney (2015); Perotti et al. (2023)), and ensures that the privately optimal investment in climate change adaptation is efficient. However, any imperfect pricing of climate risk leads to underinvestment in adaptation. Policies that contribute to the accurate pricing of climate risk in housing markets are therefore essential to encourage households to adapt optimally. Furthermore, the choice of the social discount rate plays a crucial role in determining whether the privately optimal choice of adaptation coincides with the social optimum (Stern (2007); Nordhaus (2008)).

The ability of low-income households to adapt to climate change is critically dependent on their access to finance. Since adaptation is private, households cannot credibly commit to a specific choice of adaptation. However, the liquidation value of the collateral depends directly on the extent to which borrowers adapt. When mortgage creditors take this into consideration, the size of household debt is reduced to the expected liquidation value of the *undamaged* housing capital in the next period. This credit constraint prevents constrained households from consuming the optimal level of housing capital. Hence, as adaptation only benefits households in the future, constrained households have an incentive to spend relatively less on adaptation to prevent a large reduction in their housing consumption. Due to their failure to optimally reduce their vulnerability to climate impacts, constrained households remain more exposed to its consequences. As a result, constrained households suffer a disproportionately large reduction in their housing wealth once an extreme weather event occurs, exacerbating wealth inequality. Since habitat is reduced further as climate risk rises, housing becomes relatively more important in the consumption bundle, causing constrained households to allocate a larger fraction of their budget on housing. Consequently, the investment in adaptation by low-income households becomes increasingly smaller compared to the optimal investment, translating into a widening of the 'adaptation gap' over time. As credit constraints on the household level are a significant barrier to effective adaptation (IPCC (2023); Havlinova et al. (2022)), the amplifying effects of sub-optimal adaptation on wealth inequality underscores the need for targeted policies that address the unequal impacts of climate change on those most vulnerable.

[3] I abstract from insurance polices in the present model, since insurance takes the form of monetary, ex-post compensation, rather than prevention. While this increases the resources available to households to purchase the non-durable consumption good, it neither prevents a fall in the supply of inhabitable houses, nor reduces the rate at which the supply falls, which is the dominant force at play. It further deserves mention that insurance is only available to a limited extent in practice - and becomes increasingly less available a climate risk rises.

Related literature and contribution This paper contributes to the literature that studies the incentives of households to adapt to climate change (Muis et al. (2015); Ikefuji et al. (2022); Fried (2022)). Most related to this paper is Fried (2022), who studies the macro-economic effects of climate change adaptation and the interplay between climate change adaption and federal disaster policy. Specifically, Fried (2022) shows that disaster aid policies induce moral hazard in adaptation and this reduces adaptation in the US economy, while federal subsidies for adaptation more than correct for this effect. Additionally, Fried (2022) highlights that the idiosyncratic risk component of damages caused by climate-related disasters matters for the welfare cost of climate change. This paper contributes to the literature by studying how financial incentives shape households response to adapt to climate change. Specifically, this paper examines the interplay between house price dynamics and adaptation, and the effect of credit constraints as financial friction on the efficacy of adaptation efforts.

This paper further contributes to the literature studying whether physical climate risk is priced in housing markets (Harrison et al. (2001); Bin et al. (2008); Keenan et al. (2018); Gibson et al. (2017); Bernstein et al. (2019); Bosker et al. (2019); Murfin and Spiegel (2020); Hino and Burke (2020); Baldauf et al. (2020); Bakkensen and Barrage (2021); Bakkensen et al. (2022)). While most papers find evidence that SLR risk is (at least to some extent) capitalized into US housing markets in coastal states, the empirical evidence remains mixed.[4] Consistent with the aforementioned research, this paper shows that houses exposed to climate risk trade at a discount in the market. Moreover, I show that the reduction in the supply of habitable houses due to climate-related damages increases the marginal benefit of owning a house. This effect - which is novel to the literature - increases the contemporaneous house price. I derive the condition under which this general equilibrium effect dominates. As the evolution of house prices matter for mortgage market dynamics, this paper further relates to the literature that studies the relationship between the rise in climate risk and mortgage market dynamics (Ouazad and Kahn (2019); Issler et al. (2019); Bakkensen et al. (2022)).[5] I show that households become more indebted as the impacts of climate change intensify. Moreover, I emphasize the crucial importance of adaptation from a financial stability standpoint by demonstrating its capacity to contain the rise in mortgage default rates.

Finally, this paper relates to the literature which studies the effect of response strategies to

[4]Bernstein et al. (2019), Baldauf et al. (2020) and Bakkensen and Barrage (2021) aim to identify what drives the existence and magnitude of the sea level rise discount. Bernstein et al. (2019) find that the sophistication of buyers, as well as climate change beliefs play a key role. Baldauf et al. (2020) demonstrate that exposed properties in climate change-believer neighborhoods sell at a discount compared to those in denier neighborhoods, suggesting that house prices reflect heterogeneity in beliefs about climate risks. Bakkensen and Barrage (2021) show that heterogeneity in beliefs reconciles the mixed empirical evidence and further argues that flood risk is not fully reflected in house prices due to high degrees of belief heterogeneity in coastal areas.

[5]Ouazad and Kahn (2019) show that mortgage originators are more likely to transfer default risk after natural disasters, using securitization and are more likely to increase the share of mortgages originated below the conforming loan limit after the occurrence of a natural disaster. Issler et al. (2019) show that mortgage delinquency and foreclosure rates significantly increase after a wildfire. Bakkensen et al. (2022) find that owners of houses with a larger exposure to SLR-risk are more likely to be leveraged due to heterogeneity in beliefs about climate risk. Bakkensen et al. (2022) further find that the underlying mortgage contracts have a longer maturity, and climate change pessimists are more likely to trade their climate risk exposure with banks via long-term debt contracts.

climate change on inequality (Känzig (2023); Pedroni et al. (2022)). This literature has mainly focused on the implications of climate change mitigation on economic inequality. Specifically, Känzig (2023) shows that low-income households experience a relatively larger reduction in consumption due to an increase in carbon taxes as these households (i) have a high energy share, and (ii) tend to work in sectors which are more impacted by carbon pricing policies. This paper adds to the existing literature by examining the effects of sub-optimal adaptation on wealth inequality. In particular, I demonstrate that constrained households fail to optimally reduce their vulnerability to climate impacts and, consequently, experience a disproportionately large reduction in their housing wealth once an extreme weather event occurs.

Roadmap The remainder of this paper is structured as follows: Section 2 introduces the theoretical framework. The conditions relevant for the definition of an equilibrium are derived in Section 3. Section 4 introduces adaptation to climate change and in Section 5 credit constraints are considered. Section 6 illustrates the equilibrium effects of the model by means of model simulations. Section 7 concludes.

2 Theoretical Framework

Time is discrete and denoted by $t \in \{0, 1, ..., \infty\}$. The economy is characterized by two overlapping generations, each consisting of a unit mass of households. Households derive utility from consuming housing and a non-durable consumption good, which is produced by firms. Households with some entrepreneurial talent set-up a unit mass of firms in each period. Firms operate for a single period and produce the non-durable consumption good, using physical and intangible capital, as well as labour in its production process. At the start of each period, an extreme weather event occurs, which hits a fraction of households and firms, and damages their housing and physical capital, respectively. All risk is idiosyncratic, and the economy's climate risk exposure rises deterministically over time. Households can invest in protective measures to adapt to climate change, which reduces the expected losses when hit by an extreme weather event. As firms only live for a single period, they do not adapt to climate change.

2.1 Households

Households live for two periods. When young, households purchase housing capital, denoted by L, from the old generation at a relative price p (the price of the consumption good is normalized to 1). Additionally, young households hold financial assets, and can invest in corporate- and mortgage debt, as well as firm equity. The value of corporate debt is denoted by D_f and the price of equity is denoted by e_f. Once old, households channel their savings, which consist of the proceeds from selling their house, as well as the return earned on their financial assets, to the purchase of the non-durable consumption good, denoted by c. There is an initial generation at $t = 0$, which is endowed with the supply of houses, $\bar{L}_0$.

2.1.1 Preferences

Households have preferences over housing and the non-durable consumption good, which are given by the following quasi-linearutility function

$$U\left(c_{i,t+1}, L_{i,t}\right) = c_{i,t+1} + v\left(L_{i,t}\right)$$

$v\left(L_{i,t}\right)$ captures the utility that household i obtain in period t from owning $L_{i,t}$ housing capital and $v'(\cdot) > 0, v''(\cdot) < 0$. Households maximize expected lifetime utility.

2.1.2 Labour Endowments

Households are heterogeneous in terms of skills, which are exogenously given. A fraction ϕ of households is high-skilled, h, and is endowed with $\bar{h}$ high-skilled labour. The remaining households are low-skilled, l, and are endowed with $\bar{l}$ manual labour. When young, workers supply labour inelastically in a perfectly competitive labour market and earn an income of $y_{i,t} = \{q_t \tilde{h}, w_t \tilde{l}\}$, where q_t respectively w_t denotes the high- respectively low-skilled workers' wage and $\tilde{h}$ respectively $\tilde{l}$ denote the amount of high- respectively low-skilled labour supplied (Döttling and Perotti (2017)).

2.1.3 Innovators

A fraction ε of high-skilled workers has some entrepreneurial talent. These 'innovators' set up the firms, f in the economy. Moreover, innovators create intangible capital, H, which can be viewed as the fraction of the firm's capital which is not exposed to physical climate risk. Innovators create intangible capital when young by investing $I_{H,t}$, where $I_{H,t} = H_{t+1}$. This is a costly investment, as it requires some effort cost

$$C\left(I_{H,t}\right) = \frac{\beta}{2} I_{H,t}^2$$

where β denotes the ease of innovating.

The firm operates with the intangible capital once the innovator turns old. Intangible capital is exclusively used by high-skilled workers in the production process, who attempt to capture part of the value it generates. Due to the inalienability of human capital (Hart and Moore (1994)), high-skilled partially succeed in appropriating its value. Denote the bargaining power of the innovator over the return generated on intangible capital by ω. Then, shareholders capture a fraction $(1-\omega)$ of the value intangible capital generates.

2.1.4 Climate Risk and Housing Capital

The economy is exposed to climate risk and an extreme weather event occurs in each period. Let γ_t denote the probability that a given household is hit by an extreme weather event in period, t, which increases deterministically over time. By the law of large numbers, γ_t corresponds to the fraction of households that suffer climate-related damages in any period t (Fried (2022)).

Denote by $\xi_{i,t}$ the losses suffered by a given household, i, in period, t. Losses are idiosyncratic, reflecting that extreme weather events may hit certain households harder than others.

Therefore, $\xi_{i,t}$ follows some distribution, $F\left(\xi_{i,t}\right)$, which is i.i.d. across households. Let μ_L be the expected loss as a fraction of housing capital when hit by an extreme weather event. Then, the expectation of $\xi_{i,t}$ is given by of

$$\begin{aligned}\mathbb{E}\left(\xi_{i,t}\right) &= \mathbb{E}\left(\xi_{i,t}\middle|\text{Hit by Extreme weather event}\right)\cdot\mathbb{P}\left(\text{Hit by Extreme weather event}\right)\\ &= \mu_L\gamma_t\end{aligned}$$

This reduces the housing capital owned by a given household i:

$$L_{i,t+1} = \left(1-\xi_{i,t+1}\right)L_{i,t}$$

Denote the supply of houses in a given period by $\bar{L}_t$. Then, in aggregate, the supply of houses evolves according to the following law of motion:

$$\begin{aligned}\bar{L}_{t+1} &= \int_0^1 \left(1-\xi_{i,t+1}\right)di\cdot\bar{L}_t\\ &\overset{\text{LLN}}{=}\left(1-\mu_L\gamma_{t+1}\right)\cdot\bar{L}_t\end{aligned}$$

Hence, the materialization of climate risk translates into a decline in the supply of inhabitable houses over time.

Note on the Supply of Housing It is crucial to highlight that, in the absence of climate risk, the supply of houses were to remain constant over time. Therefore, the evolution of the housing stock specifically pertains to the residual factor, which is the change in supply which is attributable to climate change and reflects the premise that certain areas become uninhabitable as climate risk materializes (Burzyński et al. (2019)). Although in reality one could respond to this by constructing new properties, this does not alter the underlying premise that there is a diminishing amount of space available for the overall population to share.

2.2 Firms

Innovators set up $\frac{1}{\phi\epsilon}$ identical firms, which means that there is a unit mass of firms in the economy in each period. Firms operate a single period and maximize profits. Firms employ both high- and low-skilled workers as well as physical and intangible capital in the production process.

2.2.1 Production Technology

Each firm produces a non-durable consumption good and use physical and intangible capital in the production process. Physical capital (K), which is exposed to climate risk, is complementary to low-skilled labour (l, see Krusell et al. (2000) and Känzig (2023), while intangible capital (H) is complementary to high-skilled labour (h). Output, Y_t, is produced according to the following constant elasticity of substitution production technology:

$$\begin{aligned}Y_t &= A\mathscr{F}\left(H_t,h_t,K_t,l_t\right)\\ &= A\left[\eta\left(H_t^{\alpha}h_t^{1-\alpha}\right)^{\rho}+(1-\eta)\left(K_t^{\alpha}l_t^{1-\alpha}\right)^{\rho}\right]^{\frac{1}{\rho}}\end{aligned}$$

where A is a technology parameter, $\rho \in [0,1)$ is the substitution parameter, and η is a distribution parameter reflecting the relative productivity of intangible capital and high-skilled labour.[6]

2.2.2 Capital Investments

Intangible capital is created by innovators. Physical capital is invested in upfront, where $I_{f,K,t} = K_{f,t+1}$. Both types of capital depreciate fully (i.e. $\delta_K = \delta_H = 1$) after the production period. Firms operating in $t = 0$ are endowed with an initial stock of physical capital, K_0, and the $t = 0$ old innovators are endowed with an initial stock of intangible capital, H_0.

2.2.3 Firm Financing

To finance the investment in physical capital each firm f issues corporate debt, with face value $D_{f,t}$. Corporate debt earns a risk-free rate of return, r_t, and is repaid each period.[7] Innovators also issues equity, which is backed by the value of the share of intangible capital appropriated by equity holders. The price of a share of firm f is denoted by $e_{f,t}$, and the quantity of shares of each firm normalized to 1. Households can trade in shares, which receive a dividend payment, $d_{f,t}$, at the end of the period.

2.2.4 Climate Risk and Firm Capital

Firms are exposed to climate change, as extreme weather events destroy its physical capital (Acharya et al. (2022)).[8] Let γ_t also capture the probability that a given firm is hit by an extreme weather event in period, t. Denote by $\xi_{f,t}$ the losses suffered by given firm, f, in period, t. Losses are again idiosyncratic and $\xi_{f,t}$ follows some distribution, $G(\xi_{f,t})$, which is i.i.d. across firms. As idiosyncratic risk is perfectly diversifiable, the expectation of $\xi_{f,t}$ is what matters to investors. Let μ_K denote the average loss as a fraction of physical capital. Then, the expectation of $\xi_{f,t}$ is given by

$$\begin{aligned}\mathbb{E}(\xi_{f,t}) &= \mathbb{E}\left(\xi_{f,t} \middle| \text{Hit by Extreme weather event}\right) \cdot \mathbb{P}(\text{Hit by Extreme weather event}) \\ &= \mu_K \gamma_t\end{aligned}$$

and the damages reduce the amount of physical capital owned by firm f which has productive value

$$\tilde{K}_t = (1 - \xi_{f,t}) K_t$$

[6] To ensure that wages of high-skilled workers are higher than those of low-skilled workers, I assume that high-skilled labour is relatively scarce

$$\frac{\phi}{1-\phi} \leq \frac{\eta}{1-\eta}$$

(see Döttling and Perotti (2017)).

[7] I abstract from corporate default in the baseline model. This implies that corporate debt is, in essence, equivalent to equity. In this context, the rate of return, r_t, can be viewed as the risk-adjusted rate of return.

[8] Acharya et al. (2022) show that tangible industries (e.g. construction, mining, oil & gas, utilities, manufactuing and forestry & fishery) are more exposed to physical climate risk than service industries.

Then, the firm's production is given by[9]

$$\tilde{Y}_t = A\Big[\eta \left(H_t^{\alpha} h_t^{1-\alpha}\right)^{\rho} + (1-\eta)\left(\tilde{K}_t^{\alpha} l_t^{1-\alpha}\right)^{\rho}\Big]^{\frac{1}{\rho}}$$

where[10]

$$\tilde{Y}_t = A\mathscr{F}(H_t, h_t, \tilde{K}_t, l_t), \qquad \mathscr{F}'_{\gamma}(H_t, h_t, \tilde{K}_t, l_t) \leq 0$$

Note on the Climate Risk Exposure of Firms vs Households There is a crucial distinction between the climate risk exposure of firms as opposed to households. Since firms only operate during one period and its capital stock depreciates fully, climate-related damages only affect the capital stock in a given period (i.e. climate risk has a *flow* effect). In contrast, the supply of housing (i.e. the stock of habitable land) shrinks permanently over time as a result of climate-related damages to housing capital (i.e. climate risk has a *stock* effect).

2.3 Financial Markets and Credit Risk

Households purchase shares, $s_{i,f,t}$ and lend to the firm, $D_{i,f,t}$ as well as to other households on the financial market. Lending only occurs against collateral and takes the form of mortgage debt backed by housing capital or corporate debt backed by physical capital.

2.3.1 Housing Market Dynamics

There is one housing market on which all home purchases and sales take place. The market opens after the extreme weather takes place, which occurs at the start of the period. Define $S_{i,t}$ as the net savings of a young household, i in period t, after the purchase of housing capital as well as shares of the firm, i.e.

$$S_{i,t} = y_{i,t} - p_t L_{i,t} - s_{i,t} e_t$$

Then, depending on whether $S_{i,t}$ is positive or negative, a given household i is classified as net lender or borrower.

$$S_{i,t} \begin{cases} \geq 0 & \text{net lender} \\ < 0 & \text{net borrower} \end{cases}$$

[9]E.g. Nordhaus (1992) and Golosov et al. (2014)) introduce climate-related damages to production using a damage function that rises in temperatures and reduces TFP. In this paper, climate-related damages to production take the form of a physical capital loss specifically. However, and without loss of generality, the production function can be rewritten to a form in which climate-related damages reduce firm's overall productivity of physical inputs (i.e. low-skilled labor and physical capital). This also capture any decline in the productivity of manual labor due to climatic impacts (Acharya et al. (2022)).

[10]Fatica et al. (2022) show that floods have a significant and long-term negative impact on firm performance, since they deteriorate firms' assets, both in the medium- and long-run. The authors find that non-exposed firms are positively effected, however, which suggests that a relocation of economic activity to less exposed areas takes place. This paper abstracts from relocation (as there is only one region), and from firm-level adaptation more generally (as firms only operate in a single period).

As housing capital forms the collateral backing the loan, and destroyed housing capital has zero liquidation value, borrowers risk default and hence pay the risky rate of return, $\hat{r}_t > r_t$.[11]

Default occurs when the proceeds from selling the undamaged housing capital is smaller than the (absolute) value of the amount borrowed , including interest:

$$p_{t+1}L_{i,t+1} \leq -(1+\hat{r}_{t+1})(S_{i,t})$$

Define the loan-to-value ratio as

$$LTV_{i,t+1} = \frac{(1+\hat{r}_{t+1})(-S_{i,t})}{p_{t+1}L_{i,t}}$$

This implicitly defines the threshold of climate damages above which a homeowner defaults as

$$\hat{\xi}_{i,t+1} = 1 - LTV_{i,t+1}$$

Then, the probability of default is given by

$$\chi_{i,t} = \mathbb{P}\left(\xi_{i,t+1} \geq \hat{\xi}_{i,t+1}\right) = \left(1 - F\left(\hat{\xi}_{i,t+1}\right)\right)$$

3 Equilibrium

3.1 Household Optimization Problem

Households maximize utility subject to the budget constraint and limited liability constraint:

$$\begin{aligned}
\max_{c_{i,t+1},L_{i,t},s_{i,t},S_{i,t}} \quad & \mathbb{E}\left(U(c_{i,t+1},L_{i,t})\right) = \mathbb{E}_t\left(c_{i,t+1}\right) + v\left(L_{i,t}\right) \\
s.t. \quad & y_{i,t} \leq p_t L_{i,t} + s_{i,t}e_t + S_{i,t} \\
& c_{i,t+1} \leq \max\{y_{i,t+1} + p_{t+1}(1-\xi_{i,t+1})L_{i,t} + d_{t+1}s_{i,t} + (1+\hat{r}_{t+1})S_{i,t}, 0\} \\
& c_{i,t+1}, L_{i,t} \geq 0,
\end{aligned}$$

where $c_{i,t+1}$ is the consumption of household i in period $t+1$, and $\mathbb{E}_t$ denotes expectations formed at date t.

3.1.1 Optimal Demand for Housing Capital

The optimal demand for housing capital in a given period t determines its price:

Lemma 1. *The demand for housing capital of each household i in period t is given by*

$$L_t^* = v'^{-1}\left((1+r_{t+1})p_t - \left(1-\mu_L\gamma_{t+1}\right)p_{t+1}\right)$$

Then, the price of housing capital in a given period, t, becomes

$$p_t = \frac{(1-\mu_L\gamma_{t+1}p_{t+1}) + v'\left(L_t^*\right)}{(1+r_{t+1})}$$

[11] Mortgage creditors have perfect foresight on the rise in climate risk and hence anticipate the effect of climate-related damages on the value of the pledged housing capital. In Section 5, I show what the implications are for results if mortgage creditors take this into account by introducing credit constraints.

The price of housing capital today is equal to the discounted value of the benefits from owning it, which consists of the marginal benefit of owning housing capital, $v'\left(L_t^*\right)$, as well as the revenue from selling the undamaged housing capital in the next period. This revenue falls in next period's *(i.e., future)* climate risk, γ_{t+1}, which reflects that houses exposed to climate risk traded at a discount in the market (see e.g. Bernstein et al. (2019); Bosker et al. (2019); Baldauf et al. (2020)).

3.1.2 Optimal Demand for Shares and Corporate Debt

The price of a share follows households' demand for share holdings, $s_{i,t}$ and is equal to the discounted value of the dividend payment, d_{t+1}

$$e_t = \frac{d_{t+1}}{(1+r_{t+1})}$$

The investment in corporate and household debt follows as residual. Households with net savings lend to others households and firms. Households with negative savings take out a mortgage.

3.2 Firm Optimization problem

Firms maximize the value to its equity holders. Since firms only operate for one period, and pay out all profits, the maximization problem is given by

$$\max_{H_t,h_t,K_t,l_t} \pi_{f,t} = \tilde{Y}_t(A, H_t, h_t, \tilde{K}_t, l_t) - \omega R_t H_t - q_t h_t - (1+r_t)D_t - w_t l_t$$

3.2.1 High- and Low-Skilled Workers' Wage

Labour markets are perfectly competitive, which means that high-skilled and low-skilled workers earn their marginal productivity.

Lemma 2. *Wages of high- and low-skilled workers, q_t respectively w_t are equal to*

$$q_t^* = A^\rho(1-\alpha)\eta \frac{\tilde{Y}_t^{1-\rho}}{h_t^{1-(1-\alpha)\rho}} H_t^{\alpha\rho}$$
$$w_t^* = A^\rho(1-\alpha)(1-\eta) \frac{\tilde{Y}_t^{1-\rho}}{l_t^{1-(1-\alpha)\rho}} \left(1-\mu_K\gamma_t\right)^{\alpha\rho} K_t^{\alpha\rho}$$

and the wage ratio, defined as $\frac{q_t^*}{w_t^*}$, is given by

$$\frac{q_t^*}{w_t^*} = \frac{\eta}{1-\eta} \cdot \left(\frac{H_t}{\left(1-\mu_K\gamma_t\right)K_t}\right)^{\alpha\rho} \cdot \left(\frac{l_t}{h_t}\right)^{1-(1-\alpha)\rho}$$

As income falls in climate-related damages, this reduces the wage of both high- and low-skilled workers. However, the loss of physical capital reduces the productivity of low-skilled workers - and consequently their wages - disproportionately. While firms reduce investments in both physical and intangible capital, firms scale back investments in physical capital relatively more due to its exposure to climate-related damages. This suppresses wages of low-skilled workers further.

Proposition 1. *Wage inequality rises in climate-related damages.*

Proof: See Appendix A.1

3.2.2 Return on Physical Capital

Firms are financially unconstrained and borrow up to the point where the marginal costs of capital are equal to its marginal productivity

Lemma 3. *The return to physical capital is given by*

$$(1+r_t^*) = A^\rho \alpha (1-\eta) \frac{\tilde{Y}_t^{1-\rho}}{\left(\left(1-\mu_K \gamma_t\right) K_t\right)^{1-\alpha\rho}} l_t^{(1-\alpha)\rho}$$

and firms fully finance the investment in physical capital by debt in each period, $I_{K,t}^* = D_t$.

Climate-related damages reduce income, and hence consumption and investment. While this suppresses the return on capital, physical capital becomes more scarce as a result of extreme weather events, which raises its price. As the elasticity of physical capital with respect to climate related-damages is larger than the elasticity of income, the latter effect dominates. Consequently, the costs of borrowing *rise* in climate-related damages.

Proposition 2. *The costs of borrowing rises in climate-related damages.*

Proof: See Appendix A.2

3.2.3 Return on Intangible Capital

Competitive firms pay a return on intangible capital equal to its marginal productivity

Lemma 4. *The return on intangible capital is given by:*

$$R_t^* = A^\rho \alpha \eta \frac{\tilde{Y}_t^{1-\rho}}{H_t^{1-\alpha\rho}} h_t^{(1-\alpha)\rho}$$

and the amount of intangible capital created by innovators is

$$I_t^* = \frac{\omega R_{t+1}^*}{\beta}$$

where $I_t^* = H_{t+1}^*$ *(Döttling and Perotti (2017)).*

3.2.4 Dividends and Share Prices

Share holders capture a fraction $(1-\omega)$ of the return to intangibles capital. Therefore, equilibrium dividends are equal to

$$\begin{aligned} d_t^* &= \tilde{Y}_t(H_t, \tilde{K}_t, h_t, l_t) - \left(\omega R_t^* H_t + q_t^* h_t + (1+r_t^*) K_{t-1} + w_t^* l_t\right) \\ &= (1-\omega) R_t^* H_t \end{aligned}$$

and share prices are given by the discounted value of the dividend payment.

$$e_t^* = \frac{(1-\omega) R_{t+1} H_{t+1}}{1+r_{t+1}}$$

Lemma 5. *Dividends decline in climate-related damages.*

The decline in dividends follows directly from the reduction in the firm's profitability. Combined with the increase in the costs of borrowing, this triggers a revaluation of firm's equity, suppressing its equity prices.

Proposition 3. *Share prices fall in climate risk.*

Proof: See Appendix A.3

As equity values decline, this reduces the financial wealth of shareholders. Therefore, financial asset price changes are an *equalizing* force. On the other hand, the rate at which households with positive savings accumulate wealth increases as the costs of borrowing rise. This increases wealth inequality and hence savings form an *unequalizing* force.

3.3 Equilibrium and Market Clearing

A competitive equilibrium is defined as an allocation $\{c_t^l, c_t^h, L_t^l, L_t^h, s_t^l, s_t^h, D_t^l, D_t^h, K_t, H_t, l_t, h_t\}_{t=0}^T$ and prices $\{p_t, e_t, r_t, R_t, w_t, h_t\}_{t=0}^T$ such that in each period, t, given prices

1. Households maximize lifetime utility
2. Firms maximize profits
3. Innovators choose intangible investment

and markets clear, i.e.

1. Total labour demand equals total labour supply:

$$\int_0^1 \left[h_{f,t}^d, l_{f,t}^d\right] df = \left[h^s, l^s\right]$$

2. Total housing demand equals total housing supply:

$$\int_0^1 L_{i,t}^* di = \bar{L}_t$$

3. Total share holdings equal total supply of shares:

$$\int_0^1 s_{i,t}^* di = 1$$

4. Total net savings from labour income are equal to the value of the firm's market capitalization and its corporate debt:

$$(1-\alpha)\tilde{Y}_t - p_t\bar{L}_t = e_t + D_t$$

where $(1-\alpha)\tilde{Y}_t = q_t\phi\tilde{h} + w_t(1-\phi)\tilde{l}$.

3.3.1 Labour Market Clearing

Households supply their entire labour endowment as its marginal product is strictly positive, i.e. $[h^s, l^s] = \{\phi\bar{h}, (1-\phi)\bar{l}\}$ and

$$\int_0^1 \left[h^d_{f,t}, l^d_{f,t}\right] df = \{\phi\bar{h}, (1-\phi)\bar{l}\}$$

3.3.2 Housing Market Clearing

The housing market clearing condition pins down the equilibrium price of housing capital in a given period, t

$$p_t^* = \frac{\left(1-\mu_L\gamma_{t+1}\right)p_{t+1} + v'\left(\bar{L}_t\right)}{1+r_{t+1}}$$

Forward substitution gives

$$p_t^* = \sum_{j=t}^{\infty}\left(\prod_{\tau=t}^{j}\frac{1}{1+r_{\tau+1}}\right)\left[v'\left(\bar{L}_j\right)\right]\prod_{\iota=t}^{j-1}(1-\mu_L\gamma_{\iota+1})$$

While houses exposed to climate risk face a price discount in the market, the materialization of climate risk reduces the supply of inhabitable houses. This increases the marginal benefit of owning a house and increases the contemporaneous house prices.

Proposition 4. *Let climate risk increase in all future periods by a factor $\zeta > 1$ (i.e., future climate risk is given by $\{\zeta\gamma_{t+1}, ..., \zeta\gamma_\infty\}$). At a constant rate of financing, the price of housing capital rises in ζ for*

$$\underbrace{-\frac{v''(\bar{L}_j)\cdot\bar{L}_j}{v'(\bar{L}_j)}}_{RRA} \geq 1$$

Proof: See Appendix A.4

i.e. the price of housing capital *rises* in climate risk when households are sufficiently risk-averse with respect to their consumption of housing. Moreover, as the supply of housing capital falls permanently and climate risk rises, this general equilibrium effect becomes stronger over time.

3.3.3 Financial Market Clearing

The financial market clearing condition requires the value of assets that carry savings over time (RHS) to equal the total savings from labour income (LHS). Recall that $D_t = K_t$. Then, the supply of physical capital is given by

$$K_t = (1-\alpha)\tilde{Y}_t - p_t\bar{L}_t - e_t$$

This condition further requires the total savings of labour income to be large enough to cover the purchase of the stock of houses, the firm's equity as well as its corporate debt. Therefore, at least one type of workers to have positive savings (Döttling and Perotti (2017)). Since high-skilled workers earn higher wages, they are net lenders and low-skilled are either net lenders or net borrowers. Then, the volume of mortgage credit in the economy, m, is given by

$$m_t = \max\left\{0, (1-\phi)\left(p_t\bar{L}_t - w_t\tilde{l}\right)\right\}$$

Corollary 1. *Let climate risk increase in all future periods by a factor $\zeta > 1$. Mortgage credit volumes rise in ζ when*

$$-\frac{v''(\bar{L}_j)\cdot\bar{L}_j}{v'(\bar{L}_j)} \geq 1$$

4 Adaptation to Climate Change

Households respond to the rise in climate risk exposure by implementing measures that reduce their vulnerability to extreme weather events. By adapting to climate change households reduce their vulnerability to future climatic impacts, leading to a leftward shift in the distribution of the *idiosyncratic* losses. It is crucial to highlight that climate change adaptation does not affect the probability of being hit by an extreme weather event in a given period (γ_t) nor it's evolution ($\gamma_{t+1},...,\gamma_\infty$), but rather reduces the expected loss of housing capital when hit by an extreme weather event. The primary advantage of adaptation, then, is to reduce the rate at which damages grow, and therefore the rate at which the housing stock falls.

4.1 Adaptation

Households invest in climate change adaptation when purchasing their housing capital. By adapting, households protect themselves against the climate-related damages due to an extreme weather event in period the next period, $t+1$. Denote by $x_{i,t} \in [0,1)$ the choice of adaptation of household i in period t. This requires a costly investment, of $\psi(x_{i,t}) = \frac{\theta}{2}L_{i,t}(x_{i,t})^2$, where θ represents the ease of adapting. The investment costs increases in the amount of housing capital, as more significant investments, such as a larger amount of storm-proof windows, are required to achieve a similar level of protection for a larger houses (Fried (2022)). Moreover, the investment costs are convex in the choice of adaptation, indicating that even the most ambitious investment in adaptation cannot prevent all climate-related loss and damages (UNEP (2022)).

Let the choice of adaptation, $x_{i,t}$, represent the fraction of expected losses (i.e. $\mu\gamma_{t+1}$) by which households reduce their idiosyncratic losses. Hence, for a given choice of adaptation, $x_{i,t}$, adaptation leads to a leftward shifts by $x_{i,t}\mu\gamma_{t+1}$ in the distribution of losses, $F(\xi_{i,t})$, of which the mean becomes

$$\mathbb{E}\left(\xi_{i,t+1}\right) = \left(1 - x_{i,t}\right)\mu_L\gamma_{t+1}$$

A given household with $x_{i,t} = 0$ does not undertake any measures to reduce idiosyncratic losses, while $x_{i,t} \to 1$ indicates that a given household has perfectly adapted to climate change and reduced nearly all expected losses. Define X_t as the total private investment in adaptation, i.e.

$$X_t = \int_0^1 x_{i,t} di$$

By its virtue of preserving housing capital, climate change adaptation reduces the rate at which the supply of inhabitable houses declines. Then, the supply of inhabitable houses evolves according to the following law of motion

$$\begin{aligned}\bar{L}_{t+1} &= \int_0^1 \left(1 - \xi_{i,t}\right) di L_t \\ &\overset{\text{LLN}}{=} \left(1 - (1 - X_t)\mu_L \gamma_{t+1}\right) \bar{L}_t\end{aligned}$$

Hence, investments in adaptation in period t prevent in part the reduction in the supply of housing in the next period, $\bar{L}_{t+1}$. Since adaptation forms an intertemporal investment and hence does not change the current supply, $\bar{L}_t$, it does not affect the utility that households obtain from owning housing capital. Rather, as adaptation raises the amount of undamaged housing capital that remains to be sold in the next period, it increases the resources available to households to purchase the non-durable consumption good when old.

Note on the Insurance I abstract from insurance polices in the present model, since insurance takes the form of monetary, ex-post compensation, rather than prevention. While this also increases the resources available to households to purchase the non-durable consumption good, it neither prevents a fall in the supply of inhabitable houses, nor reduces the rate at which the supply falls, which is the dominant force at play. While of lesser significance, it is worth noting too that under the present specification of preferences, there is no role for insurance since households are risk-neutral with respect to their consumption of the non-durable consumption good.[12]

4.2 Equilibrium with Adaptation

4.2.1 Household Optimization Problem

When households adapt to climate change, their household maximization problem is given by

[12]It further deserves mention that insurance is only available to a limited extent in practice - and becomes increasingly less available a climate risk rises. In the Netherlands, for example, 60% of land area may be affected by overflowing rivers and roughly 70% of population lives in this densely populated area. Although insurance has added value from a societal point of view, the limited options for diversification make it too expensive for any individual insurer to offer insurance against flooding due to the breach of primary flood defences. Specifically, the damage burden from any breach of primary flood defenses is potentially unmanageable and may even lead to failure of the insurer if the risk were to materialize (Doll et al. (2022)). Under the 'Wet Tegemoetkoming Schade bij Rampen' (Wts) the government may take up damage that cannot reasonably be insured. However, as the anticipation of government relief may induce moral hazard, the Minister of Justice and Security only decides whether to declare Wts applicable after the extreme weather event has occurred. This introduces uncertainty on extent to which the government covers damages, and limits willingness to provide insurance (Doll et al. (2022)).

$$\max_{c_{i,t+1},x_{i,t},L_{i,t},S_{i,t},s_{i,t}} \mathbb{E}\left(U(c_{i,t+1},L_{i,t})\right) = \mathbb{E}_t\left(c_{i,t+1}\right) + v\left(L_{i,t}\right)$$

$$s.t. \quad y_{i,t} \leq \left(p_t + \frac{\theta}{2}x_{i,t}^2\right)L_{i,t} + s_{i,t}e_t + S_{i,t}$$

$$c_{i,t+1} \leq \max\left\{y_{i,t+1} + p_{t+1}\left(1-\xi_{i,t+1}\right)L_{i,t} + d_{t+1}s_{i,t} + \left(1+\hat{r}_{t+1}\right)S_{i,t}, 0\right\}$$

$$c_{i,t+1}, x_{i,t}, L_{i,t} \geq 0$$

4.2.2 Optimal Demand for Housing and Adaptation

Climate change adaptation affects the demand for housing capital in two different directions. As investments in adaptation absorb part of households' savings, this reduces demand for housing. On the other hand, the reduction in idiosyncratic losses increases the fraction of undamaged housing capital that can be sold in the next period, which raises demand.

Lemma 6. *When households adapt to climate change, the demand for housing capital of a given household, i, in a given period, t, is given by*

$$L_t^* = v'^{-1}\left((1+r_{t+1})\left(p_t + \frac{\theta}{2}x_{i,t}^{*2}\right) - \left(1-(1-x_{i,t}^*)\mu_L\gamma_{t+1}\right)p_{t+1}\right)$$

and the price of housing capital in a given period, t, is given by

$$p_t = \frac{\left(1-(1-x_{i,t}^*)\mu_L\gamma_{t+1}\right)p_{t+1} + v'(L_t^*)}{(1+r_{t+1})} - \frac{\theta}{2}x_{i,t}^{*2}$$

This trade-off between the present costs and future benefits of adaptation is reflected in the house price. In this case, the discounted value of the benefits from owning a house equals the total per unit amount spent on housing capital, $p_t + \frac{\theta}{2}x_{i,t}^{*2}$. Hence, the investment costs directly reduce the price of housing capital, while the reduction in climate risk exposure decreases the discount at which the house is traded in the next period.

Households invest in adaptation as long as its marginal benefits outweigh its marginal cost.

Lemma 7. *The privately optimal choice of adaptation for each household i is given by*

$$x_{i,t}^* = \frac{\mu_L\gamma_{t+1}\cdot p_{t+1}}{\theta\cdot(1+r_{t+1})}$$

The optimal choice of adaptation increases in households' climate risk exposure $\left(\gamma_{t+1}\right)$ as well as the fraction of housing capital lost when hit by an extreme weather event, μ_L, as both increase the expected damages to the house. Therefore, the rise in climate exposure rises creates stronger incentives to invest in adaptation over time. Additionally, the optimal choice of adaptation increases increases in the house price, as this increases the value that is potentially lost when an extreme weather event occurs. Importantly, the privately optimal choice of adaptation is equal to the social optimum. In particular, when an unconstrained social planner maximizes utilitarian welfare, i.e.

$$\max_{x_{S,t}} \sum_{t=0}^{\infty}\left(\prod_{\tau=1}^{t}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{t+1})\frac{\theta}{2}x_{S,t}^2\bar{L}_t + v(\bar{L}_t)\right]$$

subject to

$$\bar{L}_j = \bar{L}_t \prod_{\iota=t}^{j-1} \left(1-(1-x_{S,\iota})\mu_L \gamma_{\iota+1}\right)$$

Then

$$x^*_{i,t} = x^*_{S,t}$$

This occurs as climate change adaptation increases the amount of undamaged housing capital that can be sold to the next generation. Since climate risk (and, consequently, adaptation) is accurately priced in the market, households internalize the benefits of their adaptive efforts for future generations, which prevents "tragedy of the horizon" effects (Carney (2015)).

Proposition 5. *The privately optimal investment of climate change adaptation is efficient.*

Proof: See Appendix A.5

When climate risk is not properly reflected in market pricing (e.g. as a result of heterogeneity in the beliefs about climate change), prices fail to signal the risks faced by households. Consequently, households underinvest in adaptation measures. This highlights the importance of the accurate pricing of climate risk in housing markets, as this incentivizes households to adapt optimally. Moreover, the choice of the social discount rate plays a crucial role in determining whether the privately optimal choice of adaptation coincides with the social optimum (Stern (2007); Nordhaus (2008)). Specifically, if the social planner maximizes utilitarian welfare and weights generations based on market discount rates, the private optimum and social optimum coincide. If, on the other hand, the welfare of future generations is valued at a rate lower than the market discount rate (i.e. a higher discount factor), markets no longer signal the correct value of adaptation to households, due to which underinvestment follows.

Corollary 2. *When the social planner discounts the welfare of future generations at rate $r^{SP} \in [0,1]$ and $\left(\frac{1}{1+r^{SP}}\right)^{j-t} \geq \prod_{\tau=t+1}^{j}\left(\frac{1}{1+r_{\tau+1}}\right)$, households* underinvest *in adaptation. The difference between the social and privately optimal choice of adaptation is given by*

$$\sum_{j=t+1}^{\infty}\left(\left(\frac{1}{1+r^{SP}}\right)^{j-t} - \prod_{\tau=t+1}^{j}\left(\frac{1}{1+r_{\tau+1}}\right)\right)\left[-(1+r_{j+1})\frac{\theta}{2}x_j^2 + v'(L_j)\right]\prod_{\iota=t+1}^{j-1}\left(1-(1-x_\iota)\mu_L\gamma_{\iota+1}\right)$$

Proof: See Appendix C.1

4.2.3 Equilibrium and Market Clearing

A competitive equilibrium is defined as an allocation $\{c^l_t, c^h_t, L^l_t, L^h_t, x^l_t, x^h_t, s^l_t, s^h_t, D^l_t, D^h_t, K_t, H_t, l_t, h_t\}_{t=0}^T$ and prices $\{p_t, e_t, r_t, R_t, w_t, q_t\}_{t=0}^T$ such that given prices

1. Households maximize lifetime utility
2. Firms maximize profits
3. Innovators choose intangible investment

and markets clear. Climate change adaptation affects the housing market clearing, which is now given by

$$\int_0^1 L_{i^*,t} di = \bar{L}_t$$

where $\bar{L}_t = (1-(1-X_t)\mu\gamma_t)\bar{L}_{t-1}$.

The clearing condition pins down the equilibrium price of a house in period t:

$$\begin{aligned} p_t^* &= \frac{\left(1-(1-X_t^*)\mu_L\gamma_{t+1}\right)p_{t+1} + v'(\bar{L}_t)}{(1+r_{t+1})} - \frac{\theta}{2}X_t^{*,2} \\ &= \sum_{j=t}^{\infty}\left(\prod_{\tau=t}^{j}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{j+1})\frac{\theta}{2}X_j^{*,2} + v'(\bar{L}_j)\right]\prod_{\iota=t}^{j-1}\left(1-(1-X_i)\mu_L\gamma_{\iota+1}\right) \end{aligned}$$

which reveals that climate change adaptation limits the rise in house prices by reducing the speed at which supply of inhabitable houses falls.

As adaptation is costly, it increases the total costs associated with the purchase of housing capital and hence absorbs part of households' savings. The financial market clearing condition accounts for this, i.e.

$$(1-\alpha)Y_t - \left(p_t(X_t) + \frac{\theta}{2}X_t^{2*}\right)\bar{L}_t = e_t + D_t$$

Moreover, climate change adaptation increase the funding needs of those who borrow:

$$m_t = \max\left\{0, (1-\phi)\left(p_t(X_t) + \frac{\theta}{2}X_t^2\right)\bar{L}_t - w_t\tilde{l}\right\}$$

5 Endogenous Credit Constraints

Since borrowers are protected by limited liability, borrowers with high levels of debt may find it advantageous to strategically default on their mortgage. To prevent this, housing capital is collateralized in the baseline model. However, households are exposed to idiosyncratic climate risk and this affects the value of the pledged housing capital directly. While borrowers can limit the losses suffered by adapting to climate change, I make the assumption that the choice of adaptation is non-contractible. Particularly, since adaptation is private, households cannot credibly commit to a specific choice of adaptation. The liquidation value of the collateral depends directly on the extent to which borrowers adapt and, as mortgage creditors can anticipate the losses due to the deterministic nature of climate risk in the model, they should never allow the size of household debt (gross of interest) to exceed the expected liquidation value of the *undamaged* housing capital in the next period (Kiyotaki and Moore (1997)):

$$-(1+\hat{r}_{t+1})S_t \leq \left(1-\left(1-\mathbb{E}\left(\bar{x}_{l,t}\right)\right)\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t}$$

where $\bar{x}_{l,t}$ denotes the expectation of mortgage creditors on the choice of adaptation of mortgage borrowers. As there is no aggregate uncertainty, mortgage creditors have perfect foresight on future house prices.

Climate risk affects the credit constraint along various dimensions. Demand for mortgage credit (LHS) increases as interest rates rise and wages fall, which both tighten the constraint. A rise in *current* house prices further increases mortgage credit demand and therefore tightens the constraint, while a rise in *future* house prices increase borrowing capacity (RHS) and thus loosens the constraint. The borrowing capacity falls directly in climate risk exposure, as this reduces the amount of undamaged housing capital which can be sold in the next period. Finally, adaptation offers a countervailing force against the tightening of the constraint by reducing expected climate-damages.

5.1 Equilibrium with Credit Constraints

High-skilled households are net lenders in equilibrium and, consequently, do not face any credit constraints. When low-skilled workers are net borrowers, they maximize expected utility subject to the credit constraint, as well as the budget constraint

$$\max_{c_{l,t+1}, x_{l,t}, L_{l,t}, S_{l,t}} \mathbb{E}\left(U(c_{l,t+1}, L_{l,t})\right) = \mathbb{E}_t\left(c_{l,t+1}\right) + v\left(L_{l,t}\right)$$

$$s.t. \quad w_t \leq \left(p_t + \frac{\theta}{2} x_{l,t}^2\right) L_{l,t} + S_{l,t}$$

$$c_{l,t+1} \leq \max\left\{p_{t+1}\left(1 - \xi_{l,t+1}\right) L_{l,t} + (1 + \hat{r}_{t+1}) S_{l,t}, 0\right\}$$

$$-(1 + \hat{r}_{t+1}) S_{l,t} \leq \left(1 - (1 - \mathbb{E}\left(\bar{x}_{l,t}\right))\mu_L \gamma_{t+1}\right) p_{t+1} L_{l,t}$$

$$c_{l,t+1}, x_{l,t}, L_{l,t} \geq 0$$

5.1.1 Optimal Demand for Housing and Adaptation of Constrained, Low-Income Households

Credit constrained, low-income households have limited access to the financial resources required to fund the purchase of housing capital and their invest in adaptation. In equilibrium, low-income households borrow up to the point where the constraint binds, i.e.

Lemma 8. *The demand for housing capital of constrained, low-income households is given as function of $x_{l,t}^*$:*

$$L_{l,t}^* = \frac{(1 + r_{t+1}) w_t}{(1 + r_{t+1})\left(p_t + \frac{\theta}{2} x_{l,t}^2\right) - \left(1 - \left(1 - \mathbb{E}(\bar{x}_{l,t})\right)\mu_L \gamma_{t+1}\right) p_{t+1}}$$

and the choice of adaptation of constrained, low-skilled households, $x_{l,t}^*$, is given by

$$x_{l,t}^* = \frac{\mu_L \gamma_{t+1} \cdot p_{t+1}}{\theta \cdot (1 + r_{t+1})(1 + \lambda_t)}$$

where $\lambda_t \geq 0$ denotes the shadow price of the constraint and $\mathbb{E}(\bar{x}_{l,t}) = x_{l,t}^*$ in a symmetric equilibrium.

Proposition 6. *Credit constrained, low-income households adapt relatively less to climate change than high-income households:*

$$x_l^* < x_h^*$$

Proof: See Appendix A.6

Corollary 3. *Credit constrained, low-income households invest less in adaptation than unconstrained, high-income households:*

$$\psi_l^* < \psi_h^*$$

Credit constrained, low-income households adapt relatively less to climate change than high-income households. This means that constrained households protect a relatively smaller fraction of their housing capital, and hence remain relative more exposed to extreme weather events. This occurs as credit constraints prevent constrained households from consuming the optimal level of housing capital. As adaptation only benefits households in the future, constrained households are given an incentive to spend relatively less on adaptation to prevent a large reduction in their housing consumption today. As a result, constrained households fail to reduce vulnerability to climate impacts and, once hit by the extreme weather event, they suffer a disproportionately large reduction in the housing wealth. This exacerbates wealth inequality. Additionally, the underinvestment imposes an externality on future generations. Specifically, the sub-optimal adaptation by produces an excess reduction in the supply of housing capital. This leaves future generations with less housing capital to derive utility from and accelerates the rise in house prices. Define the adaptation gap as the investment in adaptation by constrained, low-income households relative to the optimal investment, i.e.

$$\Lambda = \frac{x_{h,t}^*}{x_{l,t}^*} = (1 + \lambda_t)$$

Proposition 7. *At a constant rate of borrowing, the adaptation gap rises in climate risk when the utility function for housing is characterized by constant relative risk aversion.*

Proof: See Appendix A.7

As climate risk rises, the adaptation gap widens. This happens due to the fall in the supply of housing capital, which increases the marginal utility from housing. Housing thus becomes relatively more important in the consumption bundle of constrained households, which is reflected by an increase in the shadow value of the constraint. Since constrained households are below the optimal consumption of housing, their demand for housing responds more strongly in response to an increase in climate risk (i.e. the elasticity of the demand for housing of low-income households' with respect to climate risk is larger than the elasticity of housing demand of high-income households when credit constraints bind). As house prices rise at the same time, constrained households allocate an increasingly larger fraction of their budget on housing, rather than adaptation.

6 Parameterization and Counterfactual Analysis

To illustrate the equilibrium effects of rising climate risk on house prices, mortgage credit volumes and mortgage default rates, I provide a parameterization based on data of the Netherlands as the example, a country with a long history in flood risk management, which is likely to face an increase in its exposure to flood risk as sea levels rise. I set one time period equal to 30 years and run the model forward from 2010 to 2100. Additionally, I conduct counterfactual analysis to demonstrate the model outcomes for different scenarios of sea level rise, based on based on a low (RCP 2.6), medium (RCP 4.5), and high (RCP 8.5) greenhouse gas concentration trajectories from the IPCC (IPCC (2013)).

6.1 Parameterization and Functional Form Specification

I use externally calibrated parameters, as well as a number of normalizations. Externally calibrated parameters are based on data of the Netherlands, and are reported as of 2010. Other parameter values are taken from Döttling and Perotti (2017).

Parameter	Description	Value	Source
A	TFP in final-good production	1	Normalization
$\tilde{h}$	Inelastic supply of high-skilled labour	40	Döttling and Perotti (2017)
$\tilde{l}$	Inelastic supply of low-skilled labour	12.5	Döttling and Perotti (2017)
$\bar{L}$	Initial stock of houses	1	Normalization
α	Capital share in final-good production	0.33	Bengtsson and Waldenström (2018)
β	Ease of innovating	1	Normalization
ϵ	Fraction of high-skilled innovators	0.05	CBS (2010)
η	Relative productivity of intangible inputs	0.6	Döttling and Perotti (2017)
θ	Ease of adapting	1	Normalization
μ_L	Fraction of damages to housing capital	1	Normalization
μ_K	Fraction of damages to tangible capital	0.7	Target $\mu_L/\mu_K = 0.7$ (Fried (2022))
ρ	Substitution parameter	0	Cobb-Douglas Production
ϕ	Fraction of high skilled labour	0.20	Van der Mooren and De Vries (2022)
ω	Bargaining power of innovators	0.9	

Table 1: *Parameter Values*

I further assume that $v(L) = ln(L)$.

6.2 The Evolution of Climate Risk

For the purpose of illustrating the effects of the model, I let γ_t represent the fraction of the currently flood safe houses that a future rise in sea levels would put at risk of flooding. To establish a relationship between this fraction and various levels of sea level rise, I use estimates of Bosker et al. (2019).[13] To approximate the evolution of γ_t under different scenarios of sea level rise, I use the Climate Scenario Tables from IPCC (2013). These tables provide projections of global mean sea level rise for every decade from 2000 to 2100, with the period 1986-2005 as the reference period.[14] Additionally, projections are reported under a low (RCP 2.6),

[13]Bosker et al. (2019) provides estimates of the number of the currently flood safe houses that a future rise in sea levels would put at risk of flooding in a best-, medium-, and worst-case scenario with sea levels rising by 24, 100 or 150 cm respectively, based on Dutch elevation data.

[14]While the sea level in the North Sea may be more relevant for the Netherlands than the global mean, the sea level in the North Sea is more erratic. At time scales of several decades, sea level at the coast of the Netherlands rises at a similar rate as global sea level, indicating that the latter serves as a valid approximation (Van den Hurk et al. (2006)).

medium (RCP 4.5, RCP 6.0) and high (RCP 8.5) greenhouse gas concentration trajectory.[15] The evolution of γ under the different RCP trajectories is depicted in Figure 1.

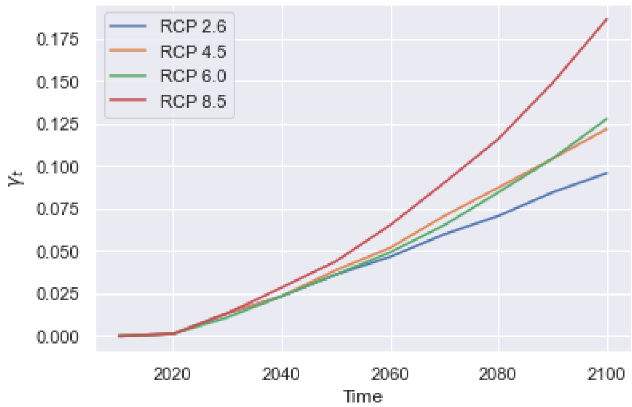

Figure 1: *The evolution of γ_t under the RCP 2.6, RCP 4.5, RCP 6.0 and RCP 8.5 trajectory.*

6.3 Results

As projections of global mean sea level rise are only available up to 2100 (IPCC (2013)), I assume that a steady-state is reached at this point in time and solve the model backwards. The results are compared to those obtained when households adapt climate change. The privately optimal choice of adaptation, x_t, is determined endogenously and the evolution of x_t is depicted in Figure 2. Counterfactual analysis is performed to compare the outcomes under different projections of the Representative Concentration Pathways (RCP) provided by IPCC (2013).

[15] The Representative Concentration Pathways trajectories describe different climate futures depending on the volume of future greenhouse gas emissions (IPCC (2014)). Under the RCP 2.6 (RCP 4.5 respectively RCP 6.0) trajectory, emissions peak in 2020 (2040 respectively 2080) and the rise in global mean temperatures is likely to stay between 0.3 to 1.7 (1.1 to 2.6 respectively 1.4 to 3.1) degrees Celsius, relative to the reference period. This translates into a rise in global mean sea levels as of 2100 of 0.26 to 0.55 (0.32 to 0.63 respectively 0.33 to 0.63) meters relative to the reference period (IPCC (2014)). Under RCP 8.5, emissions continue to rise throughout the 21^{st} century and global mean temperatures are likely to rise by approximately 2.6 to 4.8 degrees Celsius. This translates into a rise in global mean sea levels of 0.45 to 0.82 meters (IPCC (2014)).

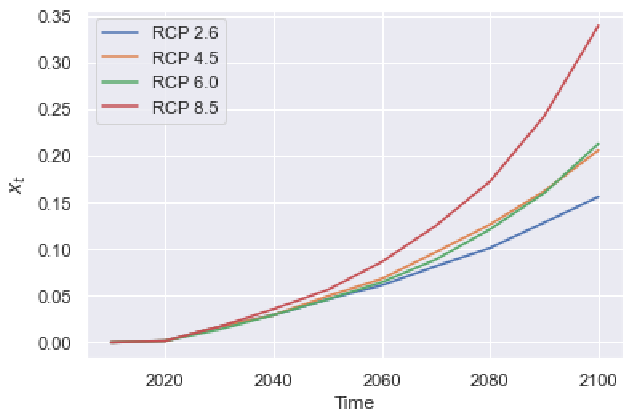

Figure 2: *The evolution of the choice of adaptation, x (right), under the RCP 2.6, 4.5, 6.0 and 8.5 trajectory.*

6.3.1 Climate Change and Adaptation

The supply of inhabitable houses falls exogenously in the model with climate risk, which leads to an increase in house prices relative to income over time, as illustrated in Figure 3. The right panel of Figure 3 illustrates the impact of households' endogenous adaptation to climate change. By investing in adaptation, households reduce the rate at which the supply falls. This weakens the marginal utility effect. Consequently, climate change adaptation reduces the rate at which house prices rise.

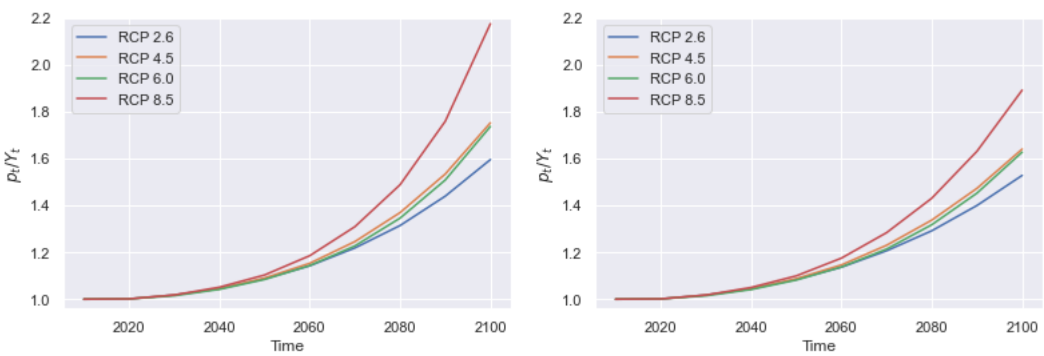

Figure 3: *The evolution of house prices to income (indexed to 1 in 2010) under the different RCP trajectories, for the model with climate change (left) and the model with adaptation to climate change (right).*

The fall in the supply of houses reduces suppresses mortgage credit demand, while the increase in house prices raises demand for credit. As wages fall, this raises demand for mortgage credit as well, causing mortgage credit to rise relative income as climate risk increases (see Figure 4).

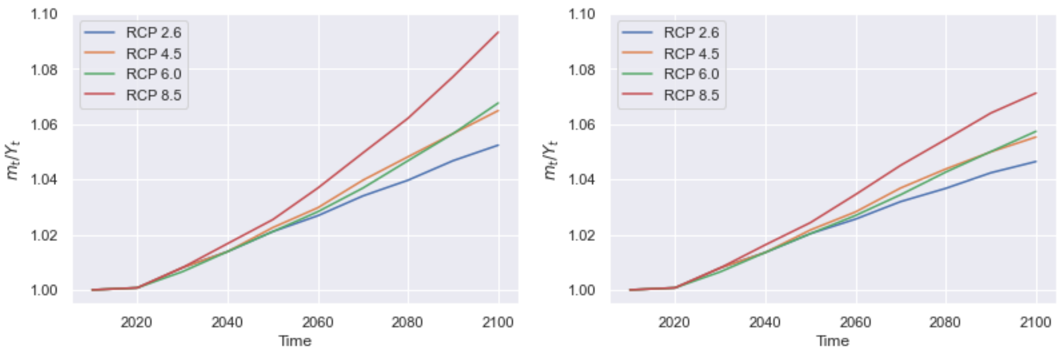

Figure 4: *The evolution of mortgage credit to income (indexed to 1 in 2010) under the different RCP trajectories, for the model with climate change (left) and the model with adaptation to climate change (right).*

As highlighted in Proposition 2, the costs of borrowing rises in the exposure to climate risk (see Figure 5). This occurs as physical capital becomes more scarce as a consequence of climate-related damages.

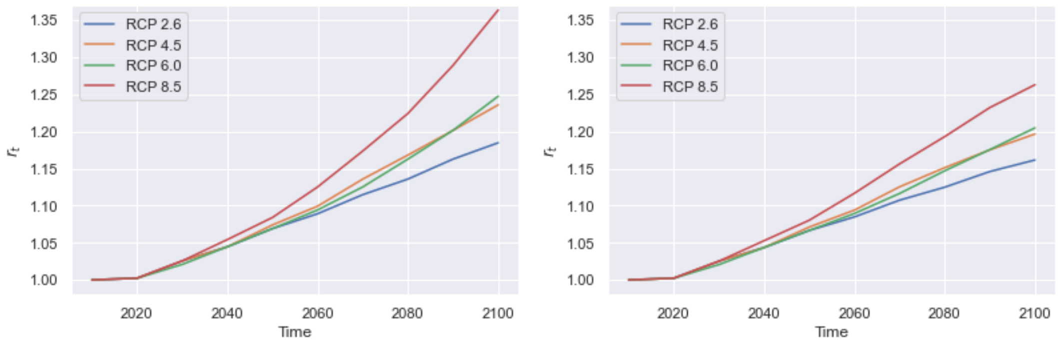

Figure 5: *The evolution of the costs of borrowing (indexed to 1 in 2010) under the different RCP trajectories, for the model with climate change (left) and the model with adaptation to climate change (right).*

When households adapt endogenously to climate change, the effect on the costs of borrowing is softened. Since adaptation limits the rise in house prices relative to income, this reduces spending on housing. Consequently, households' savings become - in part - channelled to firms and hence physical capital becomes relatively less scarce.

The simultaneous rise in mortgage credit and the costs of borrowing worsens households' financial position. This is reflected by a sharp rise in loan-to-value ratios.

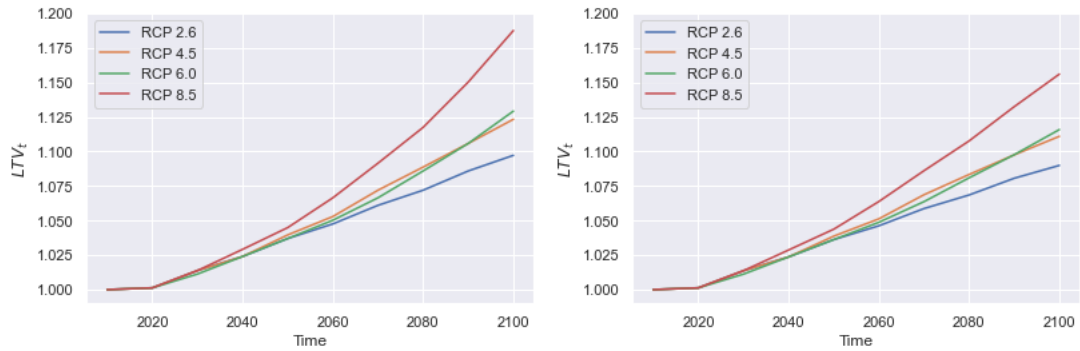

Figure 6: *The evolution of loan-to-value ratio's (indexed to 1 in 2010) under the different RCP trajectories, for the model with climate change (left) and the model with adaptation to climate change (right).*

6.3.2 Mortgage Default Rates

The rise in loan-to-value ratio's has important consequences for mortgage default. While only the expectation of ξ_{t+1} matters for pricing, the the distribution of damages must be specified to evaluate the effect of rising climate risk on mortgage default rates. I assume that $\xi_{i,t}$ follows a Beta distribution, which is defined on the interval $[0,1]$, with shape parameters $\upsilon_t \geq 0, \nu_t \geq 0$: $\xi_{i,t} \sim Beta(\upsilon_t, \nu_t)$. Given that very large losses are relatively rare, while some losses are rather common when hit by an extreme weather event, I choose the values of the shape parameters in such a way that probability mass is concentrated among low values of $\xi_{i,t}$, and large values are in the tail of the distribution. Specifically, I set $\upsilon_t = \upsilon = \mu_L$. Using that the expectation of the $Beta(\upsilon_t, \nu_t)$ distribution is defined as

$$\mathbb{E}\left(\xi_{i,t}\right) = \frac{\upsilon}{\upsilon + \nu}$$

the value of ν_t becomes

$$\nu_t^c = \frac{1}{\mu_L \gamma_t} - 1$$

Hence, this parameter is time-varying due to its dependence on γ_t. [16]

[16] Since γ_t remains relatively small under each RCP trajectory for the majority of the time period covered, ν_t becomes very large. This poses computational challenges, as the estimation of the probability density function involves the Gamma function $\Gamma(\nu_t)$, which is computationally not defined for most of the values ν_t. To overcome this, I calculate the probability density function without applying any normalization at first. Then, to ensure that the area under the curve sums up to 1, I divide each value along the curve by the integral of the "non-normalized" probability density function.

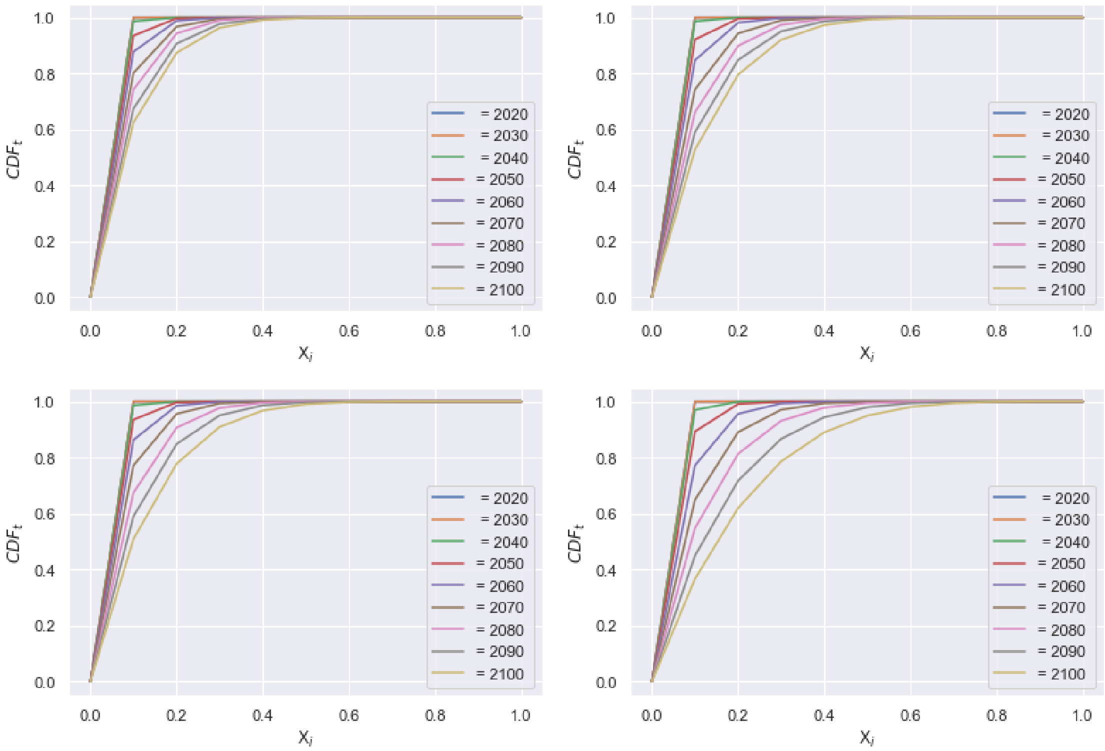

Figure 7: *The cumulative density function of $\xi_{i,t}$ for different periods in time in the model with climate change. The left upper panel plots the cumulative density functions over time under the RCP 2.6 scenario, and the right upper panel plots it under the RCP 4.5 scenario. The left lower panel plots the cumulative density functions over time under the RCP 6.0 scenario, and the right lower panel plots it under the RCP 8.5 scenario.*

The cumulative density function associated with each RCP trajectory are displayed in Figure 7. This figure shows that small losses (i.e., ξ_t) are relatively more common across all scenarios. As climate risk exposure (i.e., γ_t) rises over time, small losses become relatively less common while larger losses are suffered more frequently. This becomes evident when comparing the probability density functions across different time periods within each panel, and when comparing the probability density functions for a given time period across the different RCP trajectories.

The distribution of idiosyncratic losses is shifted to the left when households endogenously adapt to climate change. Specifically, when households adapt to climate change, the value of v_t becomes given by

$$v_t^a = \frac{1}{(1 - x_{t-1})\mu_L \gamma_t} - 1$$

and the cumulative density functions for the model with climate change adaptation are shown in Figure 8.

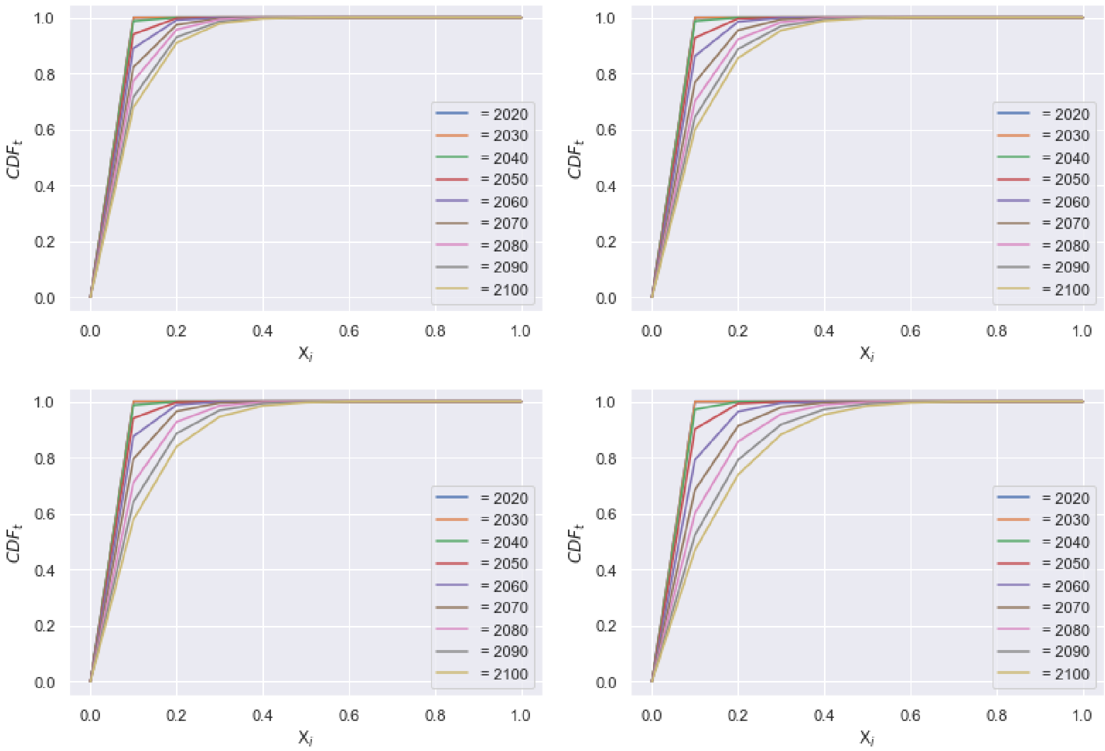

Figure 8: *The cumulative density function of $\xi_{i,t}$ for different periods in time in the model with climate change adaptation. The left upper panel plots the cumulative density functions over time under the RCP 2.6 scenario, and the right upper panel plots it under the RCP 4.5 scenario. The left lower panel plots the cumulative density functions over time under the RCP 6.0 scenario, and the right lower panel plots it under the RCP 8.5 scenario.*

The cumulative density functions are essential to the determination of mortgage default rates, which are given by

$$\chi_{i,t} = \left(1 - F\left(\hat{\xi}_{i,t+1}\right)\right)$$

where

$$\hat{\xi}_{i,t+1} = 1 - LTV_{i,t+1}$$

Figure 9 shows the evolution of mortgage default rates over time. As loan-to-value ratio's increase as climate risk rises, and larger losses become relatively more common, default rates rise over time. Moreover, mortgage default rates rise faster under more severe scenario's of climate change. Specifically, under RCP 8.5., default rates rise well above 10% at the end of the century.

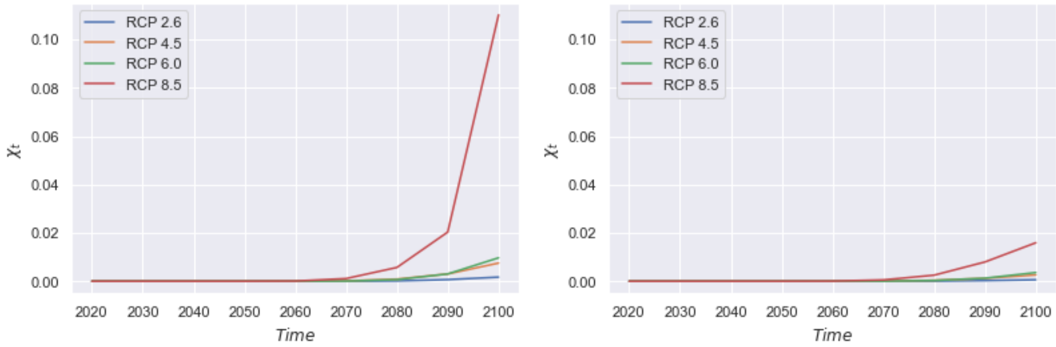

Figure 9: *The evolution of mortgage default rates under the different RCP trajectories, for the model with climate change (left) and the model with adaptation to climate change (right).*

Because climate change adaptation reduces the probability of large idiosyncratic losses (see Figure 8), and softens the rise in loan-to-value ratio's (see Figure 6), default rates are remarkably lower once households adapt to climate change. Particularly, mortgage default rates remain well below 2% throughout the century when households adapt to climate change. This underscores the crucial importance of investing adequately in measures that reduce our vulnerability to the impacts of climate change from a financial stability standpoint.

7 Conclusion

This paper explores the broader implications of climate change and adaptation, through the lens of a redistributive growth model. The analysis reveals that climate change is intrinstically redistributive. Climate-related damages reduce the productivity of low-income households disproportionately, while the destruction of capital increases the rate at which households with positive savings accumulate wealth. The ability of low-income households to adapt to climate change is critically dependent on their access to finance. Hence, in the presence of financial constraints, those most exposed to the consequences of climate risk are prevented from optimally reducing their vulnerability to climatic impacts. This amplifies wealth inequality, and the effect becomes stronger over time as as climate risk intensifies. Since credit constraints at the household level are a significant barrier to effective adaptation (IPCC (2023); Havlinova et al. (2022)), this finding underscores the need for targeted policies that address the differential impacts of climate change on those most vulnerable.

The effects of climate change and skill-biased technological potentially reinforce each other, as both trends increase the wage gap. Additionally, both climate-change and skill-biased technological change cause households to become more indebted over time and the worsening of households' financial position triggers default - even if households adapt optimally to climate change. While the analysis reveals that low-income households do not need to live in a high-risk area to be more exposed to the climate risk, a potential limitation of this analysis is that migration is not considered as adaptation mechanism. Migration is an important, yet costly, adaptation mechanism (Desmet and Rossi-Hansberg (2015); Alvarez and Rossi-

Hansberg (2021)), but also influences house price dynamics. If two regions only differ in terms of their exposure to climate risk, a higher demand for housing in the low-risk region increases the price of houses located there. Hence, migration only offers an alternative to investments in protective measures to those who are able to afford to migrate in response to climate risk. Varela Varela (2023), for example, shows that a rise in post-flood neighborhood segregation increases preexisting spatial inequities. Hence, income-based sorting could further amplify the wealth effects of climate change.

Finally, the analysis in this paper exclusively considered the response of households to physical climate risk in the form of adaptation. Due to the non-linear nature of climate risk, there are limits to which we can adapt to the climate change and prevent the potentially large losses to our economies. Moreover, the extent to which we succeed in mitigation climate change directly affects the necessity for adaptation measures. Both adaptation and mitigation measures may entail significant costs. Hence, a trade-off between two types of measures may emerge when decisions are made in the presence of financial constraints. This interplay between adaptation and mitigation remains an avenue for future research.

References

ACHARYA, V. V., T. JOHNSON, S. SUNDARESAN, AND T. TOMUNEN (2022): "Is physical climate risk priced? Evidence from regional variation in exposure to heat stress," *National Bureau of Economic Research.*

ALVAREZ, J. L. C. AND E. ROSSI-HANSBERG (2021): "The economic geography of global warming," *National Bureau of Economic Research.*

BAKKENSEN, L. A. AND L. BARRAGE (2021): "Going underwater? Flood risk belief heterogeneity and coastal home price dynamics," *The Review of Financial Studies.*

BAKKENSEN, L. A., T. PHAN, AND T.-N. WONG (2022): "Leveraging the Disagreement on Climate Change: Evidence and Theory," *FRB Richmond Working Paper.*

BALDAUF, M., L. GARLAPPI, AND C. YANNELIS (2020): "Does climate change affect real estate prices? Only if you believe in it," *Review of Financial Studies*, 33, 1256–1295.

BARRAGE, L. AND W. D. NORDHAUS (2023): "Policies, Projections, and the Social Cost of Carbon: Results from the DICE-2023 Model," *National Bureau of Economic Research.*

BAULUZ, L., F. NOVOKMET, AND M. SCHULARICK (2022): "The Anatomy of the Global Saving Glut," *CEPR Discussion Paper No. DP17215.*

BENGTSSON, E. AND D. WALDENSTRÖM (2018): "Capital shares and income inequality: Evidence from the long run," *Journal of Economic History*, 78, 712–743.

BERNSTEIN, A., M. T. GUSTAFSON, AND R. LEWIS (2019): "Disaster on the horizon: The price effect of sea level rise," *Journal of Financial Economics*, 134, 253–272.

BIN, O., J. B. KRUSE, AND C. E. LANDRY (2008): "Flood hazards, insurance rates, and amenities: Evidence from the coastal housing market," *Journal of Risk and Insurance*, 75, 63–82.

Blanchet, T. and C. Martínez-Toledano (2023): "Wealth inequality dynamics in europe and the united states: Understanding the determinants," *Journal of Monetary Economics*, 133, 25–43.

Bosker, M., H. Garretsen, G. Marlet, and C. van Woerkens (2019): "Nether Lands: Evidence on the price and perception of rare natural disasters," *Journal of the European Economic Association*, 17, 413–453.

Burzyński, M., C. Deuster, F. Docquier, and J. De Melo (2019): "Climate Change, Inequality, and Human Migration," *Journal of the European Economic Association*.

Carney, M. (2015): "Breaking the tragedy of the horizon–climate change and financial stability," *Speech given at Lloyd's of London*, 29, 220–230.

Desmet, K. and E. Rossi-Hansberg (2015): "On the spatial economic impact of global warming," *Journal of Urban Economics*, 88, 16–37.

Doll, M., S. Mattheussens, G. Van der Meulen, D. Pastoor, L. Izquierdo Rios, B. Heerma van Voss, and L. Zhang (2022): *Verzekeraars in een veranderende wereld Kansen en risico's in tijden van klimaatverandering, digitalisering en inflatie*, DNB Analyse.

Döttling, R. and E. C. Perotti (2017): "Secular trends and technological progress," *CEPR Discussion Paper No. DP12519*.

Fagereng, A., M. Gomez, E. Gouin-Bonenfant, M. Holm, B. Moll, and G. Natvik (2022): "Asset-price redistribution," Tech. rep.

Fatica, S., G. Katay, and M. Rancan (2022): "loods and firms: vulnerabilities and resilience to natural disasters in Europe," *JRC Working Paper 2022/13*.

Fried, S. (2022): "Seawalls and stilts: A quantitative macro study of climate adaptation," *The Review of Economic Studies*, 89, 3303–3344.

Gibson, M., J. T. Mullins, and A. Hill (2017): "Climate change, flood risk, and property values: Evidence from New York City," *Work. Pap., Dep. Econ., Williams Coll., Williamstown, MA*.

Golosov, M., J. Hassler, P. Krusell, and A. Tsyvinski (2014): "Optimal taxes on fossil fuel in general equilibrium," *Econometrica*, 82, 41–88.

Harrison, D., G. T. Smersh, and A. Schwartz (2001): "Environmental determinants of housing prices: the impact of flood zone status," *Journal of Real Estate Research*, 21, 3–20.

Hart, O. and J. Moore (1994): "A theory of debt based on the inalienability of human capital," *Quarterly Journal of Economics*, 109, 841–879.

Havlinova, J., B. Heerma van Voss, L. Zhang, R. Van der Molen, and F. Caloia (2022): *Financiering voor de verduurzaming van de woningvoorraad*, DNB Analyse.

Heal, G. M. and A. Millner (2014): "Agreeing to disagree on climate policy," *Proceedings of the National Academy of Sciences*, 111, 3695–3698.

HINO, M. AND M. BURKE (2020): "Does Information About Climate Risk Affect Property Values?" *NBER Working Paper No. w26807.*

IKEFUJI, M., R. J. LAEVEN, J. R. MAGNUS, AND Y. YUE (2022): "Earthquake risk embedded in property prices: Evidence from five Japanese cities," *Journal of the American Statistical Association*, 117, 82–93.

IPCC (2007): *Climate Change 2007: Impacts, Adaptation and Vulnerability. Contribution of Working Group II to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change*, Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

——— (2013): "Annex II: Climate System Scenario Tables," in *Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*, Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

——— (2014): *Climate Change 2014: Mitigation of Climate Change. Contribution of Working Group III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*, Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

——— (2021): *Climate Change 2021: The Physical Science Basis Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*, Cambridge University Press, In Press.

——— (2023): "Summary for Policymakers," in *Synthesis Report of the IPCC Sixth Assessment Report (AR6)*, Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

ISSLER, P., R. STANTON, C. VERGARA-ALERT, AND N. WALLACE (2019): "Mortgage markets with climate-change risk: Evidence from wildfires in California," *Available at SSRN 3511843.*

KAHN, M. E., K. MOHADDES, R. N. NG, M. H. PESARAN, M. RAISSI, AND J.-C. YANG (2021): "Long-term macroeconomic effects of climate change: A cross-country analysis," *Energy Economics*, 104, 105624.

KÄNZIG, D. R. (2023): "The unequal economic consequences of carbon pricing," *National Bureau of Economic Research.*

KEENAN, J. M., T. HILL, AND A. GUMBER (2018): "Climate gentrification: from theory to empiricism in Miami-Dade County, Florida," *Environmental Research Letters*, 13, 054001.

KIYOTAKI, N. AND J. MOORE (1997): "Credit cycles," *Journal of Political Economy*, 105, 211–248.

KRUSELL, P., L. E. OHANIAN, J.-V. RÍOS-RULL, AND G. L. VIOLANTE (2000): "Capital-skill complementarity and inequality: A macroeconomic analysis," *Econometrica*, 68, 1029–1053.

MARTÍNEZ-TOLEDANO, C. (2020): "House price cycles, wealth inequality and portfolio reshuffling," *WID. World Working Paper*, 2.

Muis, S., B. Güneralp, B. Jongman, J. C. Aerts, and P. J. Ward (2015): "Flood risk and adaptation strategies under climate change and urban expansion: A probabilistic analysis using global data," *Science of the Total Environment*, 538, 445–457.

Murfin, J. and M. Spiegel (2020): "Is the risk of sea level rise capitalized in residential real estate?" *Review of Financial Studies*, 33, 1217–1255.

Nordhaus, W. (2008): *A question of balance: economic models of climate change*, Yale University Press, New Haven, CT.

——— (2013): *The climate casino: Risk, uncertainty, and economics for a warming world*, Yale University Press.

Nordhaus, W. D. (1992): "An optimal transition path for controlling greenhouse gases," *Science*, 258, 1315–1319.

——— (2007): "A review of the Stern review on the economics of climate change," *Journal of economic literature*, 45, 686–702.

Ouazad, A. and M. E. Kahn (2019): "Mortgage finance in the face of rising climate risk," *NBER Working Paper No. w26322.*

Pedroni, M., T. Douenne, and A. J. Hummel (2022): "Optimal Fiscal Policy in a Second-Best Climate Economy Model with Heterogeneous Agents," *Available at SSRN 4018468.*

Perotti, E., F. Van der Ploeg, and Y. Van der Straten (2023): "Loss of Habitat and Rising Inequality: The Political Economy of Climate Change Adaptation," Tech. rep.

Saez, E. and G. Zucman (2016): "Wealth inequality in the United States since 1913: Evidence from capitalized income tax data," *The Quarterly Journal of Economics*, 131, 519–578.

Stern, N. (2007): *The economics of climate change: the Stern review*, Cambridge University Press.

UNEP (2022): "Too Little, Too Slow – Climate adaptation failure puts world at risk," in *UNEP's Adaptation Gap Report 2022*, United Nations Environmental Programme.

UNFCCC (2022): "Nationally determined contributions under the Paris Agreement. Synthesis report by the secretariat," in *Sharm el-Sheikh Climate Change Conference - November 2022*, United Nations Climate Change.

Van den Hurk, B., A. Klein Tank, G. Lenderink, A. Van Ulden, G. J. Van Oldenborgh, C. Katsman, H. Van den Brink, J. Bessembinder, W. Hazeleger, and S. Drijfhout (2006): *"KNMI Climate Change Scenarios 2006 for the Netherlands" KNMI Scientific Report WR 2006-01*, KNMI, De Bilt, The Netherlands.

Van der Mooren, F. and R. De Vries (2022): *Steeds meer hoogopgeleiden in Nederland: wat voor beroep hebben ze?*, Centraal Bureau van de Statistiek.

Varela Varela, A. (2023): "Surge of Inequality: How Different Neighborhoods React to Flooding," *Available at SSRN 4396481.*

Zucman, G. (2019): "Global wealth inequality," *Annual Review of Economics*, 11, 109–138.

Appendix A: Proof of Propositions

A.1 Proof of Proposition 1

Wage inequality increases with climate-related damages when

$$\frac{\partial\left(q_t/w_t\right)}{\partial\gamma_t} = \frac{\eta}{(1-\eta)}\cdot\left(\frac{(1-\phi)\tilde{l}}{\phi\tilde{h}}\right)^{1-(1-\alpha)\rho}\cdot\frac{\partial}{\partial\gamma_t}\left(\frac{H_t}{\left(1-\mu_K\gamma_t\right)K_t}\right)^{\alpha\rho}\geq 0$$

For this to hold, it must be that

$$\frac{\mu_L}{\left(1-\mu_K\gamma_t\right)}+\frac{\partial H_t/\partial\gamma_t}{H_t}-\frac{\partial K_t/\partial\gamma_t}{K_t}\geq 0$$

or equivalently

$$\frac{\mu_L}{\left(1-\mu_K\gamma_t\right)}\geq\frac{\partial}{\partial\gamma_t}ln\left(\frac{K_t}{H_t}\right)$$

which implies that the losses of tangible capital (*i.e. the direct effect*) should be larger than the change in investment in tangible capital relative to that of intangible capital *(i.e. the indirect effect)*.To proof this, it suffices to show that

$$\frac{\partial}{\partial\gamma_t}ln\left(\frac{K_t}{H_t}\right)\leq 0$$

Note first that

$$H_t = I^*_{t-1} = \frac{\omega}{\beta}\cdot A^\rho\alpha\eta\frac{{Y_t}^{(1-\rho)}}{H_t^{1-\alpha\rho}}h_t^{(1-\alpha)\rho}$$

Using logarithmic differentiation, the derivative of H_t to γ_t becomes

$$(2-\alpha\rho)\frac{\partial}{\partial\gamma_t}ln(H_t) = (1-\rho)\frac{\partial}{\partial\gamma_t}ln(Y_t)$$

The derivative of Y_t to γ_t is given by

$$\begin{aligned}\frac{\partial}{\partial\gamma_t}Y_t &= \frac{\partial Y_t}{\partial H_t}\cdot\frac{\partial H_t}{\partial\gamma_t}+\frac{\partial Y_t}{\partial K_t}\cdot\frac{\partial K_t}{\partial\gamma_t}\\ &= R_t\cdot\frac{\partial H_t}{\partial\gamma_t}+(1+r_t)\cdot\frac{\partial K_t}{\partial\gamma_t}\end{aligned}$$

This equation is used to find a relation between the partial derivatives of capital to climate-related damages

$$\frac{\partial K_t}{\partial\gamma_t} = \frac{\frac{(2-\alpha\rho)\cdot Y_t}{H_t\cdot(1-\rho)}-R_t}{(1+r_t)}\cdot\frac{\partial H_t}{\partial\gamma_t}$$

Now, it must be shown that

$$\frac{\partial}{\partial\gamma_t}ln\left(\frac{K_t}{H_t}\right)\leq 0\Leftrightarrow\frac{1}{H_t}\cdot\frac{\partial H_t}{\partial\gamma_t}\geq\frac{1}{K_t}\cdot\frac{\partial K_t}{\partial\gamma_t}$$

There are two cases;

1. $\partial H_t/\partial \gamma_t \leq 0$. In this case, the following condition is obtained by combining equation (81) and inequality (82)

$$R_t H_t + (1+r_t)K_t \leq \frac{(2-\alpha\rho)Y_t}{(1-\rho)}$$

Using that

$$\alpha Y_t = R_t H_t + (1+r_t)K_t$$

gives

$$\alpha \leq 2$$

which is always satisfied. Therefore,

Lemma 9. *The elasticity of tangible capital to climate-related damages is higher than the elasticity of intangible capital, i.e.*

$$\frac{1}{K_t} \cdot \left|\frac{\partial K_t}{\partial \gamma_t}\right| \geq \frac{1}{H_t} \cdot \left|\frac{\partial H_t}{\partial \gamma_t}\right|$$

and wage inequality increases in climate-related damages when $\partial H_t/\partial \gamma_t \leq 0$.

2. $\partial H_t/\partial \gamma_t \geq 0$. From Lemma 9, it follows that

Corollary 4. *The partial derivatives of H_t, K_t and Y_t to γ_t have the same sign, i.e.*

$$\frac{\partial H_t}{\partial \gamma_t} \geq 0 \implies \frac{\partial K_t}{\partial \gamma_t} \geq 0 \implies \frac{\partial Y_t}{\partial \gamma_t} \geq 0$$

Since $\partial H_t/\partial \gamma_t \geq 0 \implies \partial Y_t/\partial \gamma_t \geq 0$, this case is ruled out as $\mathscr{F}_\gamma \leq 0$.

Concluding, wage inequality increases in climate-related damages.

A.2 Proof of Proposition 2

The return to tangible capital is given by

$$(1+r_t^*) = A^\rho \alpha(1-\eta)\frac{\tilde{Y}_t^{1-\rho}}{\big(\big(1-\mu_K\gamma_t\big)K_t\big)^{1-\alpha\rho}} l_t^{(1-\alpha)\rho}$$

Using logarithmic differentiation, the derivative of r_t^* to γ_t becomes

$$\frac{\partial r_t^*}{\partial \gamma_t} = \frac{(1-\rho)}{Y_t}\cdot\frac{\partial Y_t^{net}}{\partial \gamma_t} - (1-\alpha\rho)\Big[\frac{1}{K_t}\cdot\frac{\partial K_t}{\partial \gamma_t} - \frac{\mu_K}{\big(1-\mu_K\gamma_{t+1}\big)}\Big]$$

For $\rho = 0$, this derivative becomes

$$\frac{\partial r_t^*}{\partial \gamma_t}\Big|_{\rho=0} = \frac{1}{Y_t^{net}}\cdot\frac{\partial Y_t}{\partial \gamma_t} - \frac{1}{K_t}\cdot\frac{\partial K_t}{\partial \gamma_t} + \frac{\mu_K}{\big(1-\mu_K\gamma_{t+1}\big)}$$

Recall that

$$\begin{aligned}\frac{\partial}{\partial \gamma_t}\tilde{Y}_t &= \frac{\partial \tilde{Y}_t}{\partial H_t}\cdot\frac{\partial H_t}{\partial \gamma_t} + \frac{\partial \tilde{Y}_t}{\partial K_t}\cdot\frac{\partial K_t}{\partial \gamma_t}\\ &= R_t\cdot\frac{\partial H_t}{\partial \gamma_t} + (1+r_t)\cdot\frac{\partial K_t}{\partial \gamma_t}\end{aligned}$$

and that

$$\frac{\partial K_t}{\partial \gamma_t}\Big|_{\rho=0} = \frac{\frac{2\cdot\tilde{Y}_t}{H_t} - R_t}{(1+r_t)}\cdot\frac{\partial H_t}{\partial \gamma_t} \Leftrightarrow \frac{\partial H_t}{\partial \gamma_t}\Big|_{\rho=0} = (1+r_t)\cdot\frac{\partial K_t}{\partial \gamma_t}\cdot\frac{H_t}{2\tilde{Y}_t - R_t H_t}$$

then

$$\begin{aligned}\frac{\partial r_t^*}{\partial \gamma_t}\Big|_{\rho=0} &= \frac{\partial K_t}{\partial \gamma_t}\cdot\left[(1+r_t)\cdot\left(\frac{2}{2\tilde{Y}_t - R_t H_t}\right) - \frac{1}{K_t}\right] + \frac{\mu_K}{\big(1-\mu_K\gamma_{t+1}\big)}\\ &= \frac{\partial K_t}{\partial \gamma_t}\cdot\frac{2}{K_t\cdot(2\tilde{Y}_t - R_t H_t)}\cdot\big((1+r_t)K_t + R_t H_t - \tilde{Y}_t - 1/2R_t H_t\big) + \frac{\mu_K}{\big(1-\mu_K\gamma_{t+1}\big)}\end{aligned}$$

Recall that

$$\alpha\tilde{Y}_t = R_t H_t + (1+r_t)K_t$$

Then

$$\frac{\partial r_t^*}{\partial \gamma_t}\Big|_{\rho=0} = \underbrace{\frac{\partial K_t}{\partial \gamma_t}}_{\leq 0}\cdot\underbrace{\frac{2}{K_t\cdot(2\tilde{Y}_t - R_t H_t)}}_{\geq 0}\cdot\underbrace{\big((\alpha-1)\tilde{Y}_t - 1/2R_t H_t\big)}_{\leq 0} + \underbrace{\frac{\mu_K}{\big(1-\mu_K\gamma_{t+1}\big)}}_{\geq 0} \geq 0$$

Now, $\frac{\partial r_t^*}{\partial \gamma_t}$ falls in ρ, as

$$
\begin{aligned}
\frac{\partial r_t^* / \partial \gamma_t}{\partial \rho} &= -\frac{1}{Y_t} \cdot \frac{\partial Y_t}{\partial \gamma_t} + \frac{\alpha}{K_t} \cdot \frac{\partial K_t}{\partial \gamma_t} - \frac{\alpha \mu_K}{\left(1-\mu_K \gamma_t\right)} \\
&= \frac{\partial K_t}{\partial \gamma_t} \cdot \left[\frac{\alpha}{K_t} - \frac{2(1+r_t)}{2Y_t - R_t H_t} \right] - \frac{\alpha \mu_K}{\left(1-\mu_K \gamma_t\right)} \\
&= \underbrace{\frac{\partial K_t}{\partial \gamma_t}}_{\leq 0} \cdot \underbrace{\left[\frac{2}{K_t(2Y_t - R_t H_t)} \right]}_{\geq 0} \cdot \underbrace{\left[\alpha Y_t - (1+r_t) K_t - \alpha R_t H_t + \alpha/2 \cdot R_t H_t)\right]}_{\geq 0} - \underbrace{\frac{\alpha \mu_K}{\left(1-\mu_K \gamma_t\right)}}_{\geq 0} \leq 0
\end{aligned}
$$

However, for $\rho = 1$, the derivative remains negative, i.e.

$$
\frac{\partial r_t^*}{\partial \gamma_t} = (1-\alpha)\Big[-\frac{1}{K_t} \cdot \underbrace{\frac{\partial K_t}{\partial \gamma_t}}_{\leq 0} + \underbrace{\frac{\mu_K}{(1-\mu_K \gamma_{t+1})}}_{\geq 0} \Big] \geq 0
$$

As $\rho \in [0,1)$, it holds that the costs of borrowing *rise* is climate-related damages.

A.3. Proof of Proposition 3

Share prices are given by

$$e_t^* = \frac{(1-\omega)R_{t+1}H_{t+1}}{1+r_{t+1}}$$

i.e.

$$e_t^* = (1-\omega)\cdot\frac{\eta}{(1-\eta)}\cdot\left(\frac{\tilde{h}}{\tilde{l}}\right)^{(1-\alpha)\rho}\cdot H_{t+1}^{\alpha\rho}\cdot\left(\left(1-\mu_K\gamma_{t+1}\right)K_{t+1}\right)^{(1-\alpha\rho)}$$

Using logarithmic differentiation, the derivative of e_t^* to γ_{t+1} becomes

$$\frac{\partial e_t^*}{\partial\gamma_{t+1}} = \frac{\alpha\rho}{H_{t+1}}\cdot\frac{\partial H_{t+1}}{\partial\gamma_{t+1}} + (1-\alpha\rho)\left[\frac{1}{K_{t+1}}\cdot\frac{\partial K_{t+1}}{\partial\gamma_{t+1}} - \frac{\mu_K}{\left(1-\mu_K\gamma_{t+1}\right)}\right]$$

For $\rho = 0$, this derivative becomes

$$\left.\frac{\partial e_t^*}{\partial\gamma_{t+1}}\right|_{\rho=0} = \frac{1}{K_{t+1}}\cdot\underbrace{\frac{\partial K_{t+1}}{\partial\gamma_{t+1}}}_{\leq 0} - \underbrace{\frac{\mu_K}{\left(1-\mu_K\gamma_{t+1}\right)}}_{\geq 0} \leq 0$$

Now, $\frac{\partial e_t^*}{\partial\gamma_{t+1}}$ increases in ρ, as

$$\frac{\partial e_t^*/\partial\gamma_{t+1}}{\partial\rho} = \alpha\left[\frac{\mu_K}{\left(1-\mu_K\gamma_{t+1}\right)} + \frac{1}{H_{t+1}}\cdot\frac{\partial H_{t+1}}{\partial\gamma_{t+1}} - \frac{1}{K_{t+1}}\cdot\frac{\partial K_{t+1}}{\partial\gamma_{t+1}}\right]$$

which, following the proof in Appendix A.1, is positive, since $\mu_K, \mu_L \geq 0$.

However, for $\rho = 1$, the derivative remains negative, i.e.

$$\frac{\partial e_t^*}{\partial\gamma_{t+1}} = \frac{\alpha}{H_{t+1}}\cdot\underbrace{\frac{\partial H_{t+1}}{\partial\gamma_{t+1}}}_{\leq 0} + (1-\alpha)\left[\frac{1}{K_{t+1}}\cdot\underbrace{\frac{\partial K_{t+1}}{\partial\gamma_{t+1}}}_{\leq 0} - \underbrace{\frac{\mu_K}{\left(1-\mu_K\gamma_{t+1}\right)}}_{\geq 0}\right] \leq 0$$

As $\rho \in [0,1)$, it holds that share prices *rise* is climate risk.

A.4. Proof of Proposition 4

Suppose climate risk rises in all future periods by some factor $\zeta > 1$, i.e. $\{\zeta\gamma_{t+1},...,\zeta\gamma_{\infty}\}$. Then, the price of house capital is given by

$$p_t^* = \sum_{j=t}^{\infty}\left(\prod_{\tau=t}^{j}\frac{1}{1+r_{\tau+1}}\right)\left[v'\left(\bar{L}_j\right)\right]\prod_{\iota=t}^{j-1}\left(1-\mu_L\zeta\gamma_{\iota+1}\right)$$

which, for a constant rate of financing, r, becomes

$$p_t^* = \sum_{j=t}^{\infty}\left(\frac{1}{1+r}\right)^{j-t+1}\left[v'\left(\bar{L}_j\right)\right]\prod_{\iota=t}^{j-1}\left(1-\mu_L\zeta\gamma_{\iota+1}\right)$$

Then, the FOC of p_t with respect to ζ is given by

$$\frac{\partial p_t}{\partial\zeta} = \sum_{j=t}^{\infty}\left(\frac{1}{1+r}\right)^{j-t+1}\left[\frac{\partial v'(\bar{L}_j)}{\partial\bar{L}_j}\cdot\frac{\partial\bar{L}_j}{\partial\zeta}\cdot\prod_{\iota=t}^{j-1}(1-\mu_L\zeta\gamma_{\iota+1}) + v'\left(\bar{L}_j\right)\cdot\frac{\partial}{\partial\zeta}\left(\prod_{\iota'=t}^{j-1}\left(1-\mu_L\zeta\gamma_{\iota'+1}\right)\right)\right]$$

Remark that

$$\bar{L}_j = \bar{L}_t\prod_{\iota=t}^{j-1}\left(1-\mu_L\zeta\gamma_{\iota+1}\right)$$

and

$$\frac{\partial\bar{L}_j}{\partial\zeta} = -\mu_L\bar{L}_t\sum_{\iota=t}^{j-1}\gamma_{\iota+1}\prod_{\iota'=t,\iota'\neq\iota}^{j-1}\left(1-\mu_L\zeta\gamma_{\iota'+1}\right)$$

then

$$\frac{\partial p_t}{\partial\zeta} = \sum_{j=t}^{\infty}\left(\frac{1}{1+r}\right)^{j-t+1}\left[-\frac{\partial v'(\bar{L}_j)}{\partial\bar{L}_j}\cdot\bar{L}_j - v'\left(\bar{L}_j\right)\right]\cdot\mu_L\sum_{\iota=t}^{j-1}\gamma_{\iota+1}\prod_{\iota'=t,\iota'\neq\iota}^{j-1}\left(1-\mu_L\zeta\gamma_{\iota'+1}\right)$$

This is positive when

$$-\frac{\partial v'(\bar{L}_j)}{\partial\bar{L}_j}\cdot\bar{L}_j - v'\left(\bar{L}_j\right) \geq 0$$

or, equivalently, when

$$-\frac{\frac{\partial v'(\bar{L}_j)}{\partial\bar{L}_j}\cdot\bar{L}_j}{v'\left(\bar{L}_j\right)} \geq 1$$

A.5 Proof of Proposition 5

The unconstrained social planner maximizes utilitarian welfare, i.e.

$$\max_{x_t} \sum_{t=0}^{\infty} \left(\prod_{\tau=1}^{t} \frac{1}{1+r_{\tau+1}} \right) \left[-(1+r_{t+1}) \frac{\theta}{2} x_t^2 \bar{L}_t + v(\bar{L}_t) \right]$$

subject to

$$\bar{L}_j = \bar{L}_t \prod_{\iota=t}^{j-1} \left(1-(1-x_\iota)\mu_L \gamma_{\iota+1}\right)$$

The first order condition for x_t is

$$\left(\prod_{\tau=1}^{t} \frac{1}{1+r_{\tau+1}} \right) (1+r_{t+1})\theta x_t \bar{L}_t = \sum_{j=t+1}^{\infty} \left(\prod_{\tau=1}^{j} \frac{1}{1+r_{\tau+1}} \right) \left[-(1+r_{j+1}) \frac{\theta}{2} x_j^2 + v'(\bar{L}_j) \right] \frac{\partial \bar{L}_j}{\partial x_t}$$

Using that

$$\frac{\partial \bar{L}_j}{\partial x_t} = \mu_L \gamma_{t+1} \bar{L}_t \prod_{\iota=t+1}^{j-1} \left(1-(1-x_\iota)\mu_L \gamma_{\iota+1}\right)$$

this becomes

$$(1+r_{t+1})\theta x_t = \mu_L \gamma_{t+1} \sum_{j=t+1}^{\infty} \left(\prod_{\tau=t+1}^{j} \frac{1}{1+r_{\tau+1}} \right) \left[-(1+r_{j+1}) \frac{\theta}{2} x_j^2 + v'(\bar{L}_j) \right] \prod_{\iota=t+1}^{j-1} \left(1-(1-x_\iota)\mu_L \gamma_{\iota+1}\right)$$

The first-order condition of the unconstrained household is

$$(1+r_{t+1})\theta x_t = \mu_L \gamma_{t+1} \cdot p_{t+1}$$

and the first-order condition of the unconstrained social planner is

$$(1+r_{t+1})\theta x_t = \mu_L \gamma_{t+1} \sum_{j=t+1}^{\infty} \left(\prod_{\tau=t+1}^{j} \frac{1}{1+r_{\tau+1}} \right) \left[-(1+r_{j+1}) \frac{\theta}{2} x_j^2 + v'(\bar{L}_j) \right] \prod_{\iota=t+1}^{j-1} \left(1-(1-x_\iota)\mu_L \gamma_{\iota+1}\right)$$

A necessary and sufficient condition for the privately optimal level of investment to be efficient is

$$p_{t+1} = \sum_{j=t+1}^{\infty} \left(\prod_{\tau=t+1}^{j} \frac{1}{1+r_{\tau+1}} \right) \left[-(1+r_{j+1}) \frac{\theta}{2} x_j^2 + v'(\bar{L}_j) \right] \prod_{\iota=t+1}^{j-1} \left(1-(1-x_\iota)\mu_L \gamma_{\iota+1}\right)$$

Remark that the demand for housing capital of unconstrained households is

$$L_{t+1}^* = v'^{-1} \left((1+r_{t+2}) \left(p_{t+1} + \frac{\theta}{2} x_{t+1}^2 \right) - \left(1-(1-x_{t+1})\mu_L \gamma_{t+2}\right) p_{t+2} \right)$$

Together with the housing market clearing condition, this becomes

$$p_{t+1} = \left(\frac{1}{1+r_{t+2}} \right) \left[-(1+r_{t+2}) \frac{\theta}{2} x_{t+1}^2 + v'(\bar{L}_{t+1}) + \left(1-(1-x_{t+1})\mu_L \gamma_{t+2}\right) \cdot p_{t+2} \right]$$

and forward substitution of this expression gives

$$p_{t+1} = \sum_{j=t+1}^{\infty} \left(\prod_{\tau=t+1}^{j} \frac{1}{1+r_{\tau+1}} \right) \left[-(1+r_{j+1}) \frac{\theta}{2} x_j^2 + v'(\bar{L}_j) \right] \prod_{\iota=t+1}^{j-1} \left(1-(1-x_\iota)\mu_L \gamma_{\iota+1}\right)$$

Therefore, the first-order condition of the unconstrained social planner is equivalent to the first-order condition of the unconstrained household, which implies that the market outcome is efficient.

A.6 Proof of Proposition 6

The first-order condition for L_l^* is derived from the constrained household problem as

$$-(1+r_{t+1})(1+\lambda)\left(\frac{\theta}{2}x_{l,t}^{*,2}(\lambda)+p_t\right)+(1+\lambda)\left(1-\left(1-x_{l,t}^*\right)\mu_L\gamma_{t+1}\right)p_{t+1}+v'\left(L_{l,t}^*\right)=0$$

This condition defines an implicit expression for λ_t, i.e.

$$\lambda_t=\frac{\left(1-\left(1-x_{l,t}^*(\lambda_t)\right)\mu_L\gamma_{t+1}\right)p_{t+1}+v'\left(L_{l,t}^*\right)-(1+r_{t+1})\left(\frac{\theta}{2}x_{l,t}^{*,2}(\lambda_t)+p_t\right)}{(1+r_{t+1})\left(\frac{\theta}{2}x_{l,t}^{*,2}(\lambda_t)+p_t\right)-p_{t+1}\left(1-\left(1-x_{l,t}^*(\lambda_t)\right)\mu\gamma_{t+1}\right)}$$

Since λ_t denotes the change in the optimal level of utility for loosening the constraint (and the marginal utility of owning housing is strictly positive), it holds by construction that $\lambda_t \geq 0$. What remains to be determined, is under which condition $\lambda_t = 0$.

As the denominator is strictly positive (see Proof of Lemma 8), the following condition must hold for λ_t to be zero

$$p_t=\frac{\left(1-\left(1-x_{l,t}^*(0)\right)\mu_L\gamma_{t+1}\right)p_{t+1}+v'\left(L_{l,t}^*\right)}{1+r_{t+1}}-\frac{\theta}{2}x_{l,t}^{*,2}(0)$$

Now, $\lambda_t = 0 \implies x_{l,t}^* = x_{h,t}^*$. Recall that the price of housing capital is defined as

$$p_t=\frac{\left(1-(1-x_{h,t}^*)\mu_L\gamma_{t+1}\right)p_{t+1}+v'\left(L_{h,t}^*\right)}{1+r_{t+1}}-\frac{\theta}{2}x_{h,t}^{*,2}$$

Then, a necessary and sufficient condition for the above condition to hold is that $v'\left(L_{l,t}^*\right) = v'\left(L_{h,t}^*\right)$. This implies that $L_{l,t}^* = L_{h,t}^*$, which means that the constraint doesn't bind. However, when the credit constraint binds, $L_{l,t}^* < L_{h,t}^* \implies v'\left(L_{l,t}^*\right) > v'\left(L_{h,t}^*\right)$. By contradiction, it must then be the case that $\lambda_t > 0$.

A.7 Proof of Proposition 7

To evaluate the effect of a rise in γ_{t+1} on λ_t, the expression for λ_t is first rewritten as (see Proof of Lemma 8)

$$(1+\lambda_t) = \frac{v'\left(L^*_{l,t}\right)}{v'\left(L^*_{h,t}\right) + \frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}(\lambda_t)\lambda_t^2}$$

Then, the FOC becomes

$$\frac{\partial \lambda_t}{\partial \gamma_{t+1}} = \frac{\frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}} \cdot \frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}} \cdot \left[v'\left(L^*_{h,t}\right) + \frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}\lambda_t^2\right]}{\left(v'\left(L^*_{h,t}\right) + \frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}(\lambda_t)\lambda_t^2\right)^2}$$

$$- \frac{v'\left(L^*_{l,t}\right) \cdot \left[\frac{\partial v'\left(L^*_{h,t}\right)}{\partial L^*_{h,t}} \cdot \frac{\partial L^*_{h,t}}{\partial \gamma_{t+1}} + \frac{\theta}{2} \cdot \frac{\partial (1+r_{t+1})}{\partial \gamma_{t+1}} \cdot x_l^{*2}\lambda_t^2 + \theta(1+r_{t+1}) \cdot \left(\frac{\partial x^*_{l,t}}{\partial \lambda_t} \cdot \frac{\partial \lambda_t}{\partial \gamma_{t+1}} \cdot x^*_{l,t}\lambda_t^2 + x^{*2}_{l,t}\lambda_t \cdot \frac{\partial \lambda_t}{\partial \gamma_{t+1}}\right)\right]}{\left(v'\left(L^*_{h,t}\right) + \frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}(\lambda_t)\lambda_t^2\right)^2}$$

This FOC is positive if

$$\frac{\partial \lambda_t}{\partial \gamma_{t+1}} \left[1 + v'\left(L^*_{l,t}\right) \cdot \theta(1+r_{t+1}) \cdot \left(\frac{\partial x^*_{l,t}}{\partial \lambda_t} \cdot x^*_l \lambda_t^2 + x^{*2}_{l,t}\lambda_t\right)\right]$$
$$\geq \frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}} \cdot \frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}} \cdot \left[v'\left(L^*_{h,t}\right) + \frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}\lambda_t^2\right] - v'\left(L^*_{l,t}\right) \cdot \left[\frac{\partial v'\left(L^*_{h,t}\right)}{\partial L^*_{h,t}} \cdot \frac{\partial L^*_{h,t}}{\partial \gamma_{t+1}} + \frac{\theta}{2} \cdot \frac{\partial(1+r_{t+1})}{\partial \gamma_{t+1}} \cdot x_l^{*2}\lambda_t^2\right]$$

Using that

$$\frac{\partial x^*_{l,t}}{\partial \lambda_t} = -\frac{x^*_{l,t}}{(1+\lambda_t)}$$

the LHS is rewritten

$$\frac{\partial \lambda_t}{\partial \gamma_{t+1}} \left[1 + v'\left(L^*_{l,t}\right) \cdot \theta(1+r_{t+1}) \cdot \left(\frac{\partial x^*_{l,t}}{\partial \lambda_t} \cdot x^*_l \lambda_t^2 + x^{*2}_{l,t}\lambda_t\right)\right]$$
$$= \frac{\partial \lambda_t}{\partial \gamma_{t+1}} \underbrace{\left[1 + \lambda_t v'\left(L^*_{l,t}\right) \cdot \theta(1+r_{t+1})x^2_{l,t} \cdot \left(1 - \frac{\lambda_t}{(1+\lambda_t)}\right)\right]}_{\geq 0}$$

Therefore, the LHS of the equation is positive and it remains to be evaluated whether the RHS is positive as well. To this end, the RHS is rewritten

$$\frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}} \cdot \frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}} \cdot \left[v'\left(L^*_{h,t}\right) + \frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}\lambda_t^2\right] - v'\left(L^*_{l,t}\right) \cdot \left[\frac{\partial v'\left(L^*_{h,t}\right)}{\partial L^*_{h,t}} \cdot \frac{\partial L^*_{h,t}}{\partial \gamma_{t+1}} + \frac{\theta}{2} \cdot \frac{\partial(1+r_{t+1})}{\partial \gamma_{t+1}} \cdot x_l^{*2}\lambda_t^2\right]$$

$$= \frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}} \cdot \frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}} \cdot \frac{1}{v'\left(L^*_{l,t}\right)} - \frac{\partial v'\left(L^*_{h,t}\right)}{\partial L^*_{h,t}} \cdot \frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}} \cdot \frac{1}{v'\left(L^*_{h,t}\right)}$$

$$+ \frac{\left(\frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}\lambda_t^2\right)}{v'\left(L^*_{h,t}\right)} \cdot \left[\frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}} \cdot \frac{\partial L_{l,t}}{\partial \gamma_{t+1}} \cdot \frac{1}{v'\left(L^*_{l,t}\right)} - \frac{\partial(1+r_{t+1})}{\partial \gamma_{t+1}} \cdot \frac{1}{(1+r_{t+1})}\right]$$

At a constant cost of borrowing, this becomes

$$\underbrace{\frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}}}_{\leq 0} \cdot \underbrace{\frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}}}_{\leq 0} \cdot \underbrace{\frac{1}{v'\left(L^*_{l,t}\right)}}_{\geq 0} - \underbrace{\frac{\partial v'\left(L^*_{h,t}\right)}{\partial L^*_{h,t}}}_{\leq 0} \cdot \underbrace{\frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}}}_{\leq 0} \cdot \underbrace{\frac{1}{v'\left(L^*_{h,t}\right)}}_{\geq 0} + \underbrace{\frac{\left(\frac{\theta}{2}(1+r_{t+1})x^{*2}_{l,t}\lambda^2_t\right)}{v'\left(L^*_{h,t}\right)}}_{\geq 0} \cdot \underbrace{\frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}}}_{\leq 0} \cdot \underbrace{\frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}}}_{\leq 0} \cdot \underbrace{\frac{1}{v'\left(L^*_{l,t}\right)}}_{\geq 0}$$

Then, in order for $\partial \lambda_t / \partial \gamma_{t+1}$ to be positive, it must hold that

$$\frac{\partial v'\left(L^*_{l,t}\right)}{\partial L^*_{l,t}} \cdot \frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}} \cdot \frac{1}{v'\left(L^*_{l,t}\right)} - \frac{\partial v'\left(L^*_{h,t}\right)}{\partial L^*_{h,t}} \cdot \frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}} \cdot \frac{1}{v'\left(L^*_{h,t}\right)} \geq 0$$

Suppose the utility function is characterized by CRRA with relative risk aversion coefficient ς. Then, the expression becomes

$$-\varsigma \left(\underbrace{\frac{\partial L^*_{l,t}}{\partial \gamma_{t+1}}}_{\leq 0} \cdot \frac{1}{L^*_{l,t}} - \underbrace{\frac{\partial L^*_{h,t}}{\partial \gamma_{t+1}}}_{\leq 0} \cdot \frac{1}{L^*_{h,t}} \right) \geq 0$$

which holds as the elasticity of the demand for housing of constrained households is larger than the elasticity for housing of unconstrained households in the presence of binding financial constraints. Hence, λ_t rises in γ_{t+1} if the utility function for housing is characterized by CRRA with RRA coefficient ς and the cost of borrowing is constant.

In the general equilibrium, it must additionally hold that

$$\left[\frac{\partial v'\left(L^*_{l,t}\right)}{\partial \gamma_{t+1}} \cdot \frac{1}{v'\left(L^*_{l,t}\right)} - \frac{\partial (1+r_{t+1})}{\partial \gamma_{t+1}} \cdot \frac{1}{(1+r_{t+1})} \right] \geq 0$$

while it is likely that this condition holds due to the concavity of the utility function in housing, this remains to be verified by means of model simulations.

Appendix B: Proof of Lemmas

B.1 Proof of Lemma 1

Households maximize utility subject to the budget constraint and limited liability constraint:

$$\max_{c_{i,t+1},L_{i,t},s_{i,t},S_{i,t}} \mathbb{E}\left(U(c_{i,t+1},L_{i,t})\right) = \mathbb{E}_t\left(c_{i,t+1}\right) + v\left(L_{i,t}\right)$$

$$s.t. \quad y_{i,t} \leq p_t L_{i,t} + s_{i,t}e_t + S_{i,t}$$

$$c_{i,t+1} \leq \max\left\{y_{i,t+1} + p_{t+1}\left(1-\xi_{i,t+1}\right)L_{i,t} + d_{t+1}s_{i,t} + (1+\hat{r}_{t+1})\,S_{i,t}, 0\right\}$$

$$c_{i,t+1}, L_{i,t} \geq 0,$$

where $c_{i,t+1}$ is the consumption of household i in period $t+1$ and $\mathbb{E}_t$ denotes expectations formed at date t.

Given the probability of default, the expectation of household i's consumption in period $t+1$, $c_{i,t+1}$, as formed at date t, becomes:

$$\mathbb{E}_t\left(c_{i,t+1}\right) = F\left(\hat{\xi}_{i,t+1}\right)\left(p_{t+1}\left(1-\mathbb{E}\left(\xi_{i,t+1}|\xi_{t+1} \leq \hat{\xi}_{i,t+1}\right)\right)L_{i,t} + (1+\hat{r}_{t+1})S_{i,t}\right) + d_{t+1}s_{i,t}$$

No arbitrage requires that the expected payoff of investing in corporate debt is equal to the expected payoff of holding household debt:

$$(1+r_{t+1})(-S_{i,t}) = F\left(\hat{\xi}_{i,t+1}\right)(1+\hat{r}_{t+1})(-S_{i,t}) + \left(1-F\left(\hat{\xi}_{i,t+1}\right)\right)p_{t+1}\left(1-\mathbb{E}\left(\xi_{i,t+1}|\xi_{i,t+1} > \hat{\xi}_{i,t+1}\right)\right)L_{i,t}$$

where the expected payoff of holding household debt is equal to the repayment of the loan with interest in case the household does not default and the revenue from selling the collateral in case of default.

The no-arbitrage condition can be rewritten as

$$F(\hat{\xi}_{i,t+1})\left(p_{t+1}\left(1-\mathbb{E}\left(\xi_{i,t+1}|\xi_{i,t+1} \leq \hat{\xi}_{i,t+1}\right)\right)L_{i,t} + (1+\hat{r}_{t+1})(S_{i,t})\right) = (1+r_{t+1})(S_{i,t}) + p_{t+1}\left(1-\mathbb{E}\left(\xi_{i,t+1}\right)\right)L_{i,t+1}$$

and the expectation of household i's consumption in period $t+1$, $c_{i,t+1}$, as formed at date t, becomes

$$\mathbb{E}_t\left(c_{i,t+1}\right) = (1+r_{t+1})(S_{i,t}) + p_{t+1}\left(1-\mathbb{E}\left(\xi_{i,t+1}\right)\right)L_{i,t} + d_{t+1}s_{i,t}$$

Using that $\mathbb{E}\left(\xi_{i,t+1}\right) = \mu\gamma_{t+1}$, the household optimization problem can be written as

$$\max_{c_{i,t+1},L_{i,t},s_{i,t}} \mathbb{E}\left(U(c_{i,t+1},L_{i,t})\right) = (1+r_{t+1})(S_{i,t}) + p_{t+1}\left(1-\mu_L\gamma_{t+1}\right)L_{i,t} + d_{t+1}s_{i,t} + v\left(L_{i,t}\right)$$

$$s.t. \quad y_{i,t} \leq p_t L_{i,t} + s_{i,t}e_t + S_{i,t}$$

$$c_{i,t+1}, L_{i,t} \geq 0,$$

and the budget constraint is substituted to obtain

$$\max_{c_{i,t+1},L_{i,t},s_{i,t}} \mathbb{E}\left(U(c_{i,t+1},L_{i,t})\right) = (1+r_{t+1})(y_{i,t} - p_t L_{i,t} - s_{i,t}e_t) + p_{t+1}\left(1-\mu_L\gamma_{t+1}\right)L_{i,t} + d_{t+1}s_{i,t} + v\left(L_{i,t}\right)$$

$$s.t. c_{i,t+1}, L_{i,t} \geq 0,$$

The FOC for $L_{i,t}$ is given by

$$-(1+r_{t+1})p_t + p_{t+1}\left(1-\mu_L\gamma_{t+1}\right) + v'\left(L_{i,t}\right) = 0$$

and the demand for housing capital of each household i in period t is given by

$$L_t^* = v'^{-1}\left((1+r_{t+1})p_t - p_{t+1}\left(1-\mu_L\gamma_{t+1}\right)\right)$$

and the price of housing capital in a given period, t, becomes

$$p_t = \frac{\left(1-\mu_L\gamma_{t+1}\right)p_{t+1} + v'\left(L_t^*\right)}{(1+r_{t+1})}$$

B.2 Proof of Lemma 2

The wages of high-skilled workers, q_t, follows from the FOC for h_t from the firm optimization problem:

$$\begin{aligned} q_t^* &= \frac{\partial \tilde{Y}_t}{\partial h_t} \\ &= A^\rho (1-\alpha)\eta \frac{\tilde{Y}_t^{1-\rho}}{h_t^{1-(1-\alpha)\rho}} H^{\alpha\rho} \end{aligned}$$

and the wages low-skilled workers, l_t, follow from the FOC for l_t from the firm optimization problem:

$$\begin{aligned} w_t^* &= \frac{\partial \tilde{Y}_t}{\partial l_t} \\ &= A^\rho (1-\alpha)(1-\eta) \frac{\tilde{Y}_t^{1-\rho}}{l_t^{1-(1-\alpha)\rho}} K^{\alpha\rho} \end{aligned}$$

Then, the wage ratio becomes

$$\frac{q_t^*}{w_t^*} = \frac{\eta}{(1-\eta)} \cdot \left(\frac{H_t}{\left(1-\mu_K\gamma_t\right)K_t}\right)^{\alpha\rho} \cdot \left(\frac{l_t}{h_t}\right)^{1-(1-\alpha)\rho}$$

B.3 Proof of Lemma 3

The return to tangible capital follow from the FOC for K_t from the firm optimization problem:

$$\begin{aligned} (1+r_t^*) &= \frac{\partial \tilde{Y}_t}{\partial K_t} \\ &= A^\rho \alpha (1-\eta) \frac{\tilde{Y}_t^{1-\rho}}{\left(\left(1-\mu_K\gamma_t\right)K_t\right)^{1-\alpha\rho}} l^{(1-\alpha)\rho} \end{aligned}$$

B.4 Proof of Lemma 4

The return to intangible capital follow from the FOC for H_t from the firm optimization problem:

$$R_t^* = \frac{\partial \tilde{Y}_t}{\partial H_t}$$
$$= A^{\rho}\alpha\eta \frac{\tilde{Y}_t^{1-\rho}}{H_t^{1-\alpha\rho}} h^{(1-\alpha)\rho}$$

The productive use of intangible capital requires the commitment of innovators, who capture a fraction ω of its value. This means that the return earned by innovators on the intangible capital they create, H_{t+1}, is $\omega R_{t+1} H_{t+1}$. The effort cost associated with creating H_{t+1} units of intangible capital is $C(I_{H,t+1}) = \frac{1}{2} I_{H,t+1}^2$. Innovators, then, create intangible capital until its marginal benefits are equal to its marginal costs and invest

$$I_t^* = \frac{\omega R_{t+1}^*}{\beta}$$

where $I_t^* = H_{t+1}^*$.

B.5 Proof of Lemma 5

Dividends of each firm f in period t are given by

$$d_t^* = (1-\omega) R_t^* H_t^*$$
$$= A^{\rho}\alpha\eta \tilde{Y}_t^{1-\rho} H_t^{\alpha\rho} h^{(1-\alpha)\rho}$$

Using logarithmic differentiation, the derivative of d_t^* to γ_t becomes

$$\frac{\partial d_t^*}{\partial \gamma_t} = \frac{1-\rho}{\tilde{Y}_t}\frac{\partial \tilde{Y}_t}{\partial \gamma_t} + \frac{\alpha\rho}{H_t}\frac{\partial H_t}{\partial \gamma_t}$$

which is smaller than 0.

Proof of Lemma 6

With adaptation, households' optimization problem becomes

$$\max_{c_{i,t+1}, x_{i,t}, L_{i,t}, S_{i,t}, s_{i,t}} \mathbb{E}\left(U(c_{i,t+1}, L_{i,t})\right) = \mathbb{E}_t\left(c_{i,t+1}\right) + v\left(L_{i,t}\right)$$
$$s.t. \quad y_{i,t} \le \left(p_t + \frac{\theta}{2}x_{i,t}^2\right) L_{i,t} + s_{i,t} e_t + S_{i,t}$$
$$c_{i,t+1} \le \max\left\{y_{i,t+1} + p_{t+1}\left(1-\xi_{i,t+1}\right) L_{i,t} + d_{t+1} s_{i,t} + (1+\hat{r}_{t+1}) S_{i,t}, 0\right\}$$
$$c_{i,t+1}, x_{i,t}, L_{i,t} \ge 0$$

where $c_{i,t+1}$ is the consumption of household i in period $t+1$ and $\mathbb{E}_t$ denotes expectations formed at date t.

The same no-arbitrage condition as in Appendix B.1 can be used to find the expectation, formed at date t, of household i's consumption in period $t+1$, $c_{i,t+1}$, becomes

$$\mathbb{E}_t\left(c_{i,t+1}\right) = (1+r_{t+1})(S_{i,t}) + p_{t+1}\left(1-\mathbb{E}\left(\xi_{i,t+1}\right)\right) L_{i,t} + d_{t+1} s_{i,t}$$

With adaptation $\mathbb{E}\left(\xi_{i,t+1}\right) = \left(1 - x_{i,t}\right)\mu\gamma_{t+1}$ and the household optimization problem can be written as

$$\max_{c_{i,t+1},x_{i,t},L_{i,t},S_{i,t},s_{i,t}} \mathbb{E}\left(U(c_{i,t+1},x_{i,t},L_{i,t})\right) = (1+r_{t+1})\left(S_{i,t}\right) + p_{t+1}\left(1-(1-x_{i,t})\mu_L\gamma_{t+1}\right)L_{i,t} + d_{t+1}s_{i,t} + v(L_{i,t})$$

$$s.t. \quad y_{i,t} \leq \left(p_t + \frac{\theta}{2}x_{i,t}^2\right)L_{i,t} + S_{i,t} + s_{i,t}e_t$$

$$c_{i,t+1}, x_{i,t}, L_{i,t} \geq 0$$

and the budget constraint is substituted to obtain

$$\max_{c_{i,t+1},x_{i,t},L_{i,t},S_{i,t},s_{i,t}} \mathbb{E}\left(U(c_{i,t+1},x_{i,t},L_{i,t})\right) = (1+r_{t+1})\left(y_{i,t} - \left(p_t + \frac{\theta}{2}x_{i,t}^2\right)L_{i,t} - s_{i,t}e_t\right)$$
$$+ p_{t+1}\left(1-(1-x_{i,t})\mu_L\gamma_{t+1}\right)L_{i,t} + d_{t+1}s_{i,t} + v(L_{i,t})$$
$$s.t. \quad c_{i,t+1}, x_{i,t}, L_{i,t} \geq 0$$

The FOC for $L_{i,t}$ is given by

$$-(1+r_{t+1})\left(p_t + \frac{\theta}{2}x_{i,t}^2\right) + p_{t+1}\left(1-(1-x_{i,t})\mu_L\gamma_{t+1}\right) + v'\left(L_{i,t}\right) = 0$$

and the demand for housing capital of each household i in period t is given by

$$L_t^* = v'^{-1}\left((1+r_{t+1})\left(p_t + \frac{\theta}{2}x_{i,t}^2\right) - p_{t+1}\left(1-(1-x_{i,t})\mu_L\gamma_{t+1}\right)\right)$$

and the price of housing capital in a given period, t, becomes

$$p_t = \frac{\left(1-(1-x_{i,t})\mu_L\gamma_{t+1}\right)p_{t+1} + v'\left(L_t^*\right)}{(1+r_{t+1})} - \frac{\theta}{2}x_{i,t}^2$$

B.7 Proof of Lemma 7

The optimal choice of adaptation of household i in period t follow from the FOC from the household problem in Section B.6 to $x_{i,t}$:

$$x_t^* = \frac{\mu_L\gamma_{t+1}p_{t+1}}{\theta(1+r_{t+1})}$$

B.8 Proof of Lemma 8

Low-skilled households maximize expected utility subject to their budget constraint and the credit constraint

$$\max_{c_{l,t+1},x_{l,t},L_{l,t},S_{l,t}} \mathbb{E}\left(U(c_{l,t+1},L_{l,t})\right) = \mathbb{E}_t\left(c_{l,t+1}\right) + v\left(L_{l,t}\right)$$

$$s.t. \quad w_t \leq \left(p_t + \frac{\theta}{2}x_{l,t}^2\right)L_{l,t} + S_{l,t}$$

$$c_{l,t+1} \leq \max\left\{y_{l,t+1} + p_{t+1}\left(1-\xi_{l,t+1}\right)L_{l,t} + (1+\hat{r}_{t+1})S_{l,t}, 0\right\}$$

$$-(1+\hat{r}_{t+1})S_{l,t} \leq \left(1-(1-\mathbb{E}\left(\bar{x}_{l,t}\right))\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t}$$

$$c_{l,t+1}, x_{l,t}, L_{l,t} \geq 0$$

where $c_{l,t+1}$ is the consumption of household i in period $t+1$ and $\mathbb{E}_t$ denotes expectations formed at date t.

The same no-arbitrage condition as in Appendix B.6 can be used to find the expectation, formed at date t, of low-skilled household l's consumption in period $t+1$, $c_{l,t+1}$, becomes

$$\mathbb{E}_t\left(c_{l,t+1}\right) = (1+r_{t+1})(S_{l,t}) + p_{t+1}\left(1-\mathbb{E}\left(\xi_{l,t+1}\right)\right)L_{l,t} + d_{t+1}s_{l,t}$$

Using that $\mathbb{E}\left(\xi_{l,t+1}\right) = \left(1-x_{l,t}\right)\mu\gamma_{t+1}$, the optimization problem of low-skilled households can be written as

$$\begin{aligned}\max_{c_{l,t+1},x_{l,t},L_{l,t},S_{l,t}} \mathbb{E}\left(U(c_{l,t+1},x_{l,t},L_{l,t})\right) &= (1+r_{t+1})\left(S_{l,t}\right) + p_{t+1}\left(1-(1-x_{l,t})\mu_L\gamma_{t+1}\right)L_{l,t} + v\left(L_{l,t}\right)\\ s.t.\quad w_t &\leq \left(p_t+\frac{\theta}{2}x_{l,t}^2\right)L_{l,t}+S_{l,t}\\ -(1+r_{t+1})S_{l,t} &\leq \left(1-(1-\mathbb{E}\left(\bar{x}_{l,t}\right))\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t}\\ c_{l,t+1},x_{l,t},L_{l,t} &\geq 0\end{aligned}$$

and the budget constraint is substituted to obtain

$$\begin{aligned}\max_{c_{l,t+1},x_{l,t},L_{l,t},S_{l,t}} \mathbb{E}\left(U(c_{l,t+1},x_{l,t},L_{l,t})\right) &= (1+r_{t+1})\left(w_t-\left(p_t+\frac{\theta}{2}x_{l,t}^2\right)L_{l,t}\right) + p_{t+1}\left(1-(1-x_{l,t})\mu_L\gamma_{t+1}\right)L_{l,t} + v\left(L_{l,t}\right)\\ s.t.\quad -(1+r_{t+1})S_{l,t} &\leq \left(1-(1-\mathbb{E}\left(\bar{x}_{l,t}\right))\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t}\\ c_{l,t+1},x_{l,t},L_{l,t} &\geq 0\end{aligned}$$

Define the Lagrangian for this constrained maximization problem as

$$\begin{aligned}\mathscr{L} = &(1+r_{t+1})\left(w_t-\left(p_t+\frac{\theta}{2}x_{l,t}^2\right)L_{l,t}\right) + p_{t+1}\left(1-(1-x_{l,t})\mu_L\gamma_{t+1}\right)L_{l,t} + v\left(L_{l,t}\right)\\ &+\lambda\left(\left(1-(1-\mathbb{E}\left(\bar{x}_{l,t}\right))\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t}-\left((1+r_{t+1})\left[\left(p_t+\frac{\theta}{2}x_{l,t}^{*2}\right)L_{l,t}-w_t\right]\right)\right)\end{aligned}$$

where $\lambda \geq 0$ denotes the Lagrangian multiplier.

The FOC for the demand for housing by low-skilled household l, $L_{l,t}$ is given by

$$-(1+r_{t+1})(1+\lambda)\left(p_t+\frac{\theta}{2}x_{l,t}^{*2}\right) + p_{t+1}\left(1-(1-x_{l,t}^*)\mu_L\gamma_{t+1}\right) + v'\left(L_{l,t}^*\right) + \lambda\left(1-(1-\mathbb{E}\left(\bar{x}_{l,t}\right))\mu_L\gamma_{t+1}\right)p_{t+1} = 0$$

where $\mathbb{E}(\bar{x}_{l,t}) = x_{l,t}^*$ in a symmetric equilibrium. The optimal demand for housing by low-skilled household l, $L_{l,t}^*$ is defined in terms of the optimal choice of adaptation by low-skilled household, $x_{l,t}^*$

$$L_{l,t}^* = v'^{-1}\left((1+r_{t+1})(1+\lambda)\left(p_t+\frac{\theta}{2}x_{l,t}^{*2}\right)-(1+\lambda)\left(1-\left(1-x_{l,t}^*\right)\mu_L\gamma_{t+1}\right)p_{t+1}\right)$$

As the degree to which households invest in adaptation is not contractible, the FOC for the choice of adaptation of low-skilled household, $x_{l,t}$, is given by

$$-\theta(1+r_{t+1})(1+\lambda)x_{l,t}^*L_{l,t}^* + \mu\gamma_{t+1}p_{t+1}L_{l,t}^* = 0$$

and the optimal choice of adaptation of low-skilled household, $x^*_{l,t}$, is given by

$$x^*_{l,t} = \frac{\mu \gamma_{t+1} p_{t+1}}{\theta(1+r_{t+1})(1+\lambda)}$$

The FOC for the Lagrangian multiplier, λ is given by

$$\left(1-(1-\mathbb{E}(\bar{x}_t))\mu_L \gamma_{t+1}\right) p_{t+1} L_{l,t} - \left((1+r_{t+1})\left[\left(p_t + \frac{\theta}{2} x^2_{l,t}\right) L_{l,t} - w_t\right]\right) = 0$$

where $\mathbb{E}(x_{l,t}) = x^*_{l,t}$ in a symmetric equilibrium. This equation provides the corner-solution for the optimal demand for housing by low-skilled household, $L^*_{l,t}$ in terms of their optimal choice of adaptation, $x^*_{l,t}$

$$L^*_{l,t} = \frac{(1+r_{t+1}) w_t}{(1+r_{t+1})\left(p_t + \frac{\theta}{2} x^2_{l,t}\right) - \left(1-\left(1-x^*_{l,t}\right)\mu_L \gamma_{t+1}\right) p_{t+1}}$$

As $L_{l,t} \geq 0$, it must hold that

$$(1+r_{t+1})\left(p_t + \frac{\theta}{2} x^2_{l,t}\right) - \left(1-\left(1-x^*_{l,t}\right)\mu_L \gamma_{t+1}\right) p_{t+1} > 0$$

Substituting the price of housing capital gives

$$(1+r_{t+1})\left(\frac{\left(1-(1-x^*_{h,t})\mu_L \gamma_{t+1}\right) p_{t+1} + v'\left(L^*_{h,t}\right)}{1+r_{t+1}} - \frac{\theta}{2}\left(x^{*,2}_{h,t} - x^2_{l,t}\right)\right) - \left(1-\left(1-x^*_{l,t}\right)\mu_L \gamma_{t+1}\right) p_{t+1} > 0$$

which can be rewritten to obtain the following condition

$$v'\left(L^*_{h,t}\right) > \left(x^*_{l,t} - x^*_{h,t}\right)\mu_L \gamma_{t+1} \cdot p_{t+1} - (1+r_{t+1})\frac{\theta}{2}\left(x^{*,2}_{l,t} - x^{*,2}_{h,t}\right)$$

Now, there are two cases

1. When the credit constraint doesn't bind, $x_{l,t} = x_{h,t}$. Then, the RHS becomes 0 and the condition is satisfied for any strictly positive $L_{h,t}$.

2. When the credit constraint binds (i.e. $\lambda > 0$), the expression for $x^*_{l,t}$ and $x^*_{h,t}$ can be used to rewrite the RHS to

$$\begin{aligned}
&-\frac{(\mu \gamma_{t+1} p_{t+1})^2}{(1+r)\theta}\left[\frac{1}{2} - \frac{1}{(1+\lambda_t)} + \frac{1}{2(1+\lambda_t)^2}\right] \\
&= -\frac{(\mu \gamma_{t+1} p_{t+1})^2 \cdot \lambda_t^2}{2(1+r)\theta(1+\lambda_t)^2} \\
&= -\frac{\theta}{2}(1+r_{t+1}) x^{*2}_{l,t}(\lambda_t)\lambda_t^2
\end{aligned}$$

which is negative. Therefore, the condition is again satisfied for any positive $L_{h,t}$.

Appendix C: The Discounting Debate in Climate Change

The private choice of adaptation is dynamically efficient, as long as the social planner maximizes utilitarian welfare and evaluates the welfare of future generations using the market discount rate. This brings forward "the thorniest issues in all climate-change economics: how should we compare present and future costs and benefits?" (Nordhaus (2013)). While this question received considerable attention within the environmental economic literature, disagreements about the appropriate value of the social discount rate continue to be at the heart of the climate policy debate (Heal and Millner (2014)). Specifically, there is no consensus about the choice of *r* in the dynamic social welfare function of utilitarian form, i.e.

$$\int_{t=0}^{\infty} e^{-rt} u(c(t)) dt$$

which, in discrete time, is equivalent to

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t u(c(t))$$

There are two main views on this issue. Stern (2007) argues that it is immoral to use a social discount rate based on market discount rates when evaluating the welfare of future generations and favours an a priori approach, with $r = 0.014$.[17] On the other hand, Nordhaus (2008) argues that economists have no particular expertise in what is *morally* right. To ensure that models replicate reality, Nordhaus (2007) thus advocates a market based approach with $r = 0.055$.[18] While the difference in the proposed social discount rates may appear small, these small differences lead to large disparities between the estimated social cost of carbon and consequently the recommended intensity of climate mitigation policies (Heal and Millner (2014)). This paper shows that the choice of social discount rates is also crucial to the determination of optimal adaptation policies. Specifically, if the social planner were to evaluate the welfare of future generations using a social discount rate smaller than the market discount rate (i.e. a larger discount factor), the market outcome is no longer efficient and households underinvest in adaptation.

[17] Following the Ramsey rule, the relationship between the equilibrium real return on capital, r^*, and the growth rate of the economy, g^* is given by $r^* = \varsigma + \sigma \cdot g^*$, where σ denotes the elasticity of consumption and ς denotes the generational rate of time preference (Nordhaus (2007)). Stern (2007) assumes that $\sigma = 1$, $\varsigma = 0.001$ and $g^* = 0.013$. This gives a real return of capital equal to $r^* = 0.014$..

[18] Nordhaus (2008) assumes that $\sigma = 2$, $\varsigma = 0.015$ and $g^* = 0.02$. This gives a real return on capital equal to $r^* = 0.055$.

C.1 Proof of Corollary 2

Suppose the social planner discounts the welfare of future generations at rate $r^{SP} \in [0,1]$. Then, the unconstrained social planner maximizes

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r^{SP}}\right)^t \left[-(1+r_{t+1})\frac{\theta}{2}x_t^2 L_t + v(L_t)\right]$$

subject to

$$L_j = L_t \prod_{\iota=t}^{j-1} \big(1-(1-x_i)\mu_L \gamma_{\iota+1}\big)$$

The first order condition for x_t is

$$(1+r_{t+1})\theta x_t = \mu_L \gamma_{t+1} \sum_{j=t+1}^{\infty} \left(\frac{1}{1+r^{SP}}\right)^{j-t} \left[-(1+r_{j+1})\frac{\theta}{2}x_j^2 + v'(L_j)\right] \prod_{\iota=t+1}^{j-1} \big(1-(1-x_i)\mu_L \gamma_{\iota+1}\big)$$

Hence, the private solution is not efficient for $r^{SP} \neq r_{\tau+1}$. In particular, if $r^{SP} < r_{\tau+1}$,[19] households *underinvest* in adaptation and the difference between the social and privately optimal choice of adaptation is given by

$$\frac{\mu_L \gamma_{t+1}}{\theta(1+r_{t+1})} \cdot \sum_{j=t+1}^{\infty} \left(\left(\frac{1}{1+r^{SP}}\right)^{j-t} - \prod_{\tau=t+1}^{j} \left(\frac{1}{1+r_{\tau+1}}\right)\right) \left[-(1+r_{j+1})\frac{\theta}{2}x_j^2 + v'(L_j)\right] \prod_{\iota=t+1}^{j-1} \big(1-(1-x_i)\mu_L \gamma_{\iota+1}\big)$$

[19] If $r^{SP} > r_{\tau+1}$, households *overinvest* in adaptation and difference between the social and the private optimal choice of adaptation is given by

$$\frac{\mu_L \gamma_{t+1}}{\theta(1+r_{t+1})} \cdot \sum_{j=t+1}^{\infty} \left(\prod_{\tau=t+1}^{j} \left(\frac{1}{1+r_{\tau+1}}\right) - \left(\frac{1}{1+r^{SP}}\right)^{j-t}\right) \left[-(1+r_{j+1})\frac{\theta}{2}x_j^2 + v'(L_j)\right] \prod_{\iota=t+1}^{j-1} \big(1-(1-x_i)\mu_L \gamma_{\iota+1}\big)$$

However, this case is unlikely to be relevant empirically.